M.Sc Applied Mathematics

(Effective from the admitted batch of 2021-2022)

Scheme and Syllabus

DEPARTMENT OF APPLIED MATHEMATICS
COLLEGE OF SCIENCE AND TECHNOLOGY
ANDHRA UNIVERSITY, VISAKHAPATNAM

Andhra University
COLLEGE OF SCIENCE & TECHNOLOGY
M.Sc Applied Mathematics
Course Curriculum
Choice Based Credit System
With Effect From 2021-2022 Admitted Batch

Basic Structure of the course

I-Semester			
Course code		Subjects	Number of Credits
Theory Subjects: 5 Lab : 1	21AM101	1. Real Analysis	4
	21AM102	2. ODE &Integral Equations	4
	21AM103	3. Classical Mechanics	4
	21AM104	4. Discrete Mathematical Structures	4
	21AM105	5. Programming in C	4
	21AMPR101	6. Lab: Programming in C	2

II-Semester			
	Course code	Subjects	Number of Credits
Theory Subjects : 5 Viva Voce : 1	21AM201	1. Complex Analysis	4
	21AM202	2. PDE & Integral Transforms	4
	21AM203	3. Statistics & Distribution theory	4
	21AM204	4. Elements of Elasticity and Fluid Dynamics	4
	21AM205	5. Numerical Analysis	4
	21AMVV201	6. Viva Voce	2

		III-Semester	
	Course code	Subjects	Number of Credits
Theory Subjects: 5 Lab : 1	21AM301	1. Measure Theory	4
	21AM302	2.Python Programming	4
	21AM303	3. Techniques of Applied Mathematics	4
		4. Elective – I	4
		5. Elective – II	4
		6. MOOC Course*	
	Value added course	7.Intellectual Property Rights	2
	21AMPR301	8. Lab: Python Programming	2

Candidate may choose any two electives from the below

MOOC Course* : Candidate may select MOOC Course offered by NPTEL/SWAYAM/Any Government recognized organizations.

Electives:

Course Code	Semester-III
21AM304(A)	1. Boundary Value Problems-I
21AM305(B)	2. Optimization Techniques-I
21AM306(C)	3. Relativity and Cosmology –I
21AM307(D)	4. Numerical Solutions of Partial Differential Equations-I

<table>
<tr><th colspan="3">Semester-IV</th><th></th></tr>
<tr><th colspan="2">Course code</th><th>Subjects</th><th>Number of Credits</th></tr>
<tr><td rowspan="6">Theory Subjects : 4
Project Work
Viva Voce : 1</td><td>21AM401</td><td>1. Functional Analysis</td><td>4</td></tr>
<tr><td>21AM402</td><td>2. Finite Automaton &Formal Languages</td><td>4</td></tr>
<tr><td>21AM402</td><td>3. Elective-I</td><td>4</td></tr>
<tr><td></td><td>4. Elective-II</td><td>4</td></tr>
<tr><td></td><td>5. MOOC COURSE</td><td></td></tr>
<tr><td></td><td>6. Project Work</td><td>4</td></tr>
<tr><td></td><td>Value added course</td><td>7.Research Methodology</td><td>2</td></tr>
<tr><td></td><td>21AMVV201</td><td>Viva Voce</td><td>2</td></tr>
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Candidate may choose any two electives from the below

Electives:

Course Code	Semester-IV
21AM404(A)	1. Boundary Value Problems-II
21AM405(B)	2.Optimization Techniques-II
21AM406(C)	3. Relativity and Cosmology –II
21AM407(D)	4. Numerical Solutions of Partial Differential Equations-II

MOOC Course:** Candidate may select MOOC Course offered by NPTEL/SWAYAM/Any Government recognized organizations.

PROGRAMME OUTCOMES

The program outcomes of the two year M.ScApplied Mathematics Programme :

PO1: Provide in depth knowledge on the core courses of Mathematical Physics, Applied Mathematics andComputational Mathematics, and apply the knowledge of above concepts in various fields like Physical, Biological, computational and social sciences.

PO2: Develop analytical abilities and logical thinking to formulate and solve problems that they come across, and also to inculcate innovative skills, working with groups and ethical practices to maintain harmony, and participate for the development of the society.

PO3: Enable the student to acquire the necessary problem solving skills by participating in tutorials, seminars and completing an assigned project.

PO4: Provide and develop the ability and skills to obtain employment in various organizations and national level fellowships for pursuing research if he/she intends to join a research programme in a chosen field.

PO5: Develop the necessary programming skills to acquire knowledge in developing and utilizing software confidently for the problems they undertake in an Industry/Organization in which they are employed.

PROGRAMME SPECIFIC OUTCOMES

PSO1: The two years post graduate programme offered by the department of Applied Mathematics mainly aims to strike a balance between the areas of core Mathematics and Applied & Computational Mathematics.

PSO2: The course structure is designed to enable the student to seek a teaching career in an Engineering or Polytechnic or Degree or Junior college or pursue research leading to Ph.D degree after successful completion of the course.

PSO3: It will also provide inputs for getting gainful employment in software industry, in established Scientific organizations like DRDO, CSIR, NST Labs etc and in Government /Quasi government/private sectors.

PSO4: Widen the scope for employability by including some job oriented courses in curriculum like Numerical analysis, Mathematical Statistics, Operations Research, Theory of Finite Automata, Programming Languages C++, Python and for research fields Boundary Value Problems, Operations Research, Relativity & Cosmology and Numerical Solutions of Partial Differential Equations, which will enhance the Students' capabilities for employment and research.

M.Sc. APPLIED MATHEMATICS (FIRST SEMESTER)
21AM-101: REAL ANALYSIS
(With effect from 2021-2022 Admitted Batch)
Course Code:21AM101

(A total of Ten questions to be set by selecting two questions from each unit with internal choice. Each question carries 16 marks.)

Course outcomes (COs): At the end of the course, the students will be able to

CO1: Apply the knowledge of concepts of real analysis in order to study theoretical development of different mathematical techniques and their applications.
CO2: Understand the nature of abstract mathematics and explore the concepts in further details. Extend their knowledge of real variable theory for further exploration of the subject for going into research.
CO3: Identify challenging problems in real variable theory and find their appropriate solutions.
CO4: Deal with axiomatic structure of metric spaces and generalize the concepts of sequences and series, and continuous functions in metric spaces.
CO5: Use theory of Riemann-Stieltjes integral in solving definite integrals arising in different fields of science and engineering.

Course Specific outcomes (CSOs):
CSO1: Identify challenging problems in real variable theory and find their appropriate solutions.
CSO2: Demonstrate an understanding of the theory of sequences and series, continuity, differentiation and integration; Demonstrate skills in constructing rigorous mathematical arguments.
CSO3: Use multi-variable calculus in solving problems arising in different fields of science and engineering.

Learning Outcomes(LOs): On successful completion of this course, students will be able to:
LO1: Describe the fundamental properties of the real numbers that underpin the formal development of real analysis;
LO2: Demonstrate an understanding of the theory of sequences and series, continuity, differentiation and integration;
LO3: Demonstrate skills in constructing rigorous mathematical arguments;
LO4: Apply the theory in the course to solve a variety of problems at an appropriate level of difficulty;
LO5: Demonstrate skills in communicating mathematics. Read and construct mathematical arguments and proofs.

Mapping of course outcomes with the program outcomes:

	PO1	PO2	PO3	PO4	PO5
CO1	✓	✓	✓	✓	
CO2	✓	✓	✓	✓	
CO3	✓	✓	✓	✓	
CO4	✓	✓	✓	✓	
CO5	✓	✓	✓	✓	

Unit-I

Basic Topology: Finite, countable and uncountable sets, metric spaces, compact sets, perfect sets, connected sets. Continuity: Limits of functions, continuous functions, continuity and compactness, continuity and connectedness, discontinuities, monotone functions, infinite limits and limits at infinity. (Chapters 2 and 4 of Textbook. 1).

Unit-II

The Riemann- Stieltjes integral: Linearity properties, integration by parts, change of variable, reduction to a Riemann integral, monotonically increasing integrators, Riemann's condition, comparison theorems, integrators of bounded variation. (Section 7.1 to 7.7 and 7.11 to7.15 of Textbook. 2)

Unit-III

Sufficient conditions for existence of R-S. integrals, Necessary conditions for existence of R-S integrals, Mean-value theorems for R-S integrals, integral as a function of interval, second fundamental theorem of integral calculus, second mean-value theorem for Riemann integrals. (Section 7.16 to 7.22 of Textbook. 2)

Unit-IV

Multivariable Differential Calculus: Directional derivative, total derivative, Jacobian matrix, chain rule, mean-value theorem for differentiable functions, sufficient conditions for differentiability and for equality of mixed partial derivatives, Taylor's formula for real valued functions in n real variables. (Chapter 12 of Textbook. 2).

Unit-V

Sequences and Series of functions: Uniform convergence, Uniform convergence and Continuity, Uniform convergence and Integration, Uniform convergence and Differentiation. Equicontinuous families of functions, Stone – Weierstrass theorem. (Chapter 7 of Textbook. 1)

Text Books:

1. Principles of Mathematical Analysis by Walter Rudin (3rd edition) McGraw Hill international edition, 1976.
2. Mathematical Analysis by Tom M Apostol (2nd Edition) Addison-Wesley publishing Company.

M.Sc Degree Examination

First Semester

Applied Mathematics

21AM101: Real Analysis

(Effective from the admitted batch of 2021-2022)

Time: 3 hours Maximum: 80 marks

Answer one question from each unit. All questions carry equal marks

UNIT-I

1. (a) Let S$\subset \mathbb{R}^n$. Then show that following are equivalent
   (i) S is Compact.
   (ii) S is closed and bounded.
   (iii) Every infinite subset of S has a limit point in S.

   (b) Let P be a non empty perfect set in R^k. Then show that P is uncountable.

OR

2. (a) Let $\{E_n\}, n = 1,2,3, ...$ be a sequence of countable sets, then prove that $\bigcup_{n=1}^{\infty} E_n$ is a countable set.

   (b) Prove that the function defined below is discontinuous everywhere.

$$f(x) = \begin{cases} 1, & x \text{ rational}, \\ 0, & x \text{ irrational}. \end{cases}$$

UNIT-II

3. (a) Let α is increasing on $[a, b]$. If $f \in R(\alpha)$ on $[a, b]$, then show that$f^2 \in R(\alpha)$ on $[a, b]$.

   (b) If $f \in R(\alpha)$ on $[a, b]$, then show that $\alpha \in R(f)$ and also show that $\int_a^b f(x)d\alpha(x) + \int_a^b \alpha(x)df(x) = f(b)\alpha(b) - f(a)\alpha(a)$.

OR

4. (a) If $f \in R(\alpha), g \in R(\alpha)$ on $[a, b]$, then show that $C_1 f + C_2 g \in R(\alpha)$ where C_1 and C_2 are constants on $[a, b]$.

   (b) Let α is increasing on $[a, b]$. Then for any two partitions P_1 and P_2 , prove that $L(P_1, f, \alpha) \leq U(P_2, f, \alpha)$.

UNIT-III

5. (a) If $f \in R$ and $g \in R$ on $[a, b]$, let $F(x) = \int_a^x f(t)dt$, and $G(x) = \int_a^x g(t)dt$ for $x \in [a, b]$. Then show that F and G are continuous functions of bounded variation on $[a, b]$. Also show that $f \in R(G)$, $g \in R(F)$ and $\int_a^b f(x)g(x)dx = \int_a^b f(x)dG(x) = \int_a^b g(x)dF(x)$.

   (b) State and prove the mean value theorem for Riemann- Stieltjes integrals.

OR

6. (a) If f is continuous on $[a, b]$ and α is of bounded variation on $[a, b]$, then Show that $f \in R(\alpha)$ on $[a, b]$.

   (b) Let $f \in R[a, b]$ and α is continuous on $[a, b]$ with $\alpha' \in R[a, b]$, then show that the integrals $\int_a^b f(x)d\alpha(x)$, $\int_a^b f(x)\alpha'(x)dx$ exist and are equal.

UNIT-IV

7. (a) Let one of the partial derivatives $D_1 f, \dots, D_n f$ exists at c and the remaining $n - 1$ partial derivatives exist in some n-ball $B(c)$ and are continuous at c, then show that f is differentiable at c.

   (b) Find the second order Taylor expansion of $f(x, y) = e^{-(x^2+y^2)}$ about the point (1, 2).

OR

8. (a) Let u and v be two real valued functions defined on a subset S of the complex plane. Assume that u, v are differentiable at an Interior point c of S and that the partial derivatives satisfy the Cauchy-Riemann equations at c. Then show that function $f = u + iv$ has a derivative at c and $f'(c) = D_1 u(c) + iD_1 v(c)$.

   (b) Compute the directional derivative of the function $f(x, y) = x^2 y^3 + 2x^4 y$ at the point (1, -2) in the direction of the vector $u = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$. What is the maximum value of the directional derivative.

UNIT-V

9. (a) Show that there exists a real continuous function on the real line which is no where differentiable.

   (b) Let K be a compact metric space, if $\{f_n\} \in C(K)$ for $n = 1,2,3, \dots$ and if $\{f_n\}$ converges uniformly on K, then prove that $\{f_n\}$ is equicontinuous on K.

OR

10. (a) Define pointwise convergence and uniform convergence for a sequence of functions $\{(f_n), n = 1,2,3, \dots\}$. Test the convergence of $f_n(x) = \frac{(x+n)^2}{n^2}$, $x \in R$.

(b) State and prove Stone-Weierstrass theorem. **M.Sc. APPLIED MATHEMATICS (FIRST SEMESTER)**

21AM 102: **ORDINARY DIFFERENTIAL EQUATIONS & INTEGRAL EQUATIONS**

(With effect from 2021-2022 Admitted Batch)

Course Code:21AM102

((A total of Ten questions to be set by selecting two questions from each unit with internal choice. Each question carries 16 marks.)

Course Outcome:

CO1: The theory for the solution of linear differential equations with variable coefficients and reducing the order of the equations to find basis will be familiar.

CO2: To be familiar with Series solution of the equations , regular singular points and their solutions.

CO3: Establishing the existence and uniqueness of solutions of initial value problems by method of successive approximations , system of ordinary differential equations and their solutions.

CO4: Integral equations and their relation with differential equation, solution of non-homogeneous Volterra’s integral equation are taught.

CO5: To understand iterated kernels as well as Fredholm's integral equations.

Course specific outcome:

CSO1: To develop the methods to solve ODEs.

CSO2: To equip the students to solve differential equations using successive approximations along with Lipschitz condition.

CSO3: To familiarize the students to integral equations and their solutions.

Learning outcomes: On successful completion of this course, students will be able to:

LO1: get familiar with different methods to solve ordinary differential equations.

LO2: find series solutions as well as regular singular points.

LO3: use Lipschitz condition and successive approximations.

LO4: formulate integral equations and classification.

LO5: Fredholm's ,Volterra's type integral equations and various methods to solve them.

Mapping of course outcomes with the program outcomes:

	PO1	PO2	PO3	PO4	PO5
CO1	✓	✓	✓	✓	
CO2	✓	✓	✓		
CO3	✓		✓	✓	
CO4	✓	✓	✓	✓	
CO5	✓	✓		✓	

Unit-I

Linear equations with variable coefficients, the Wronskian and linear independence, reduction of the order of homogeneous equations, the non-homogeneous equations. Homogeneous equations with analytic coefficients. (Chapter 3 (excluding section 8 & 9) of Text book 1).

Unit-II

Linear equations with regular singular points, Euler's equations, series solutions, regular singular points at infinity, introduction to existence and uniqueness of solutions of 1^{st} order equations, equations with the variables separated, Exact equations. (Chapter 4 (excluding sections 5, 7 & 8)), (sections 1, 2 & 3 of chapter 5)) of Text book 1.

Unit-III

Picard’s method of successive approximations, Lipschitz condition, convergence of the successive approximations, systems as vector equations, existence & uniqueness of solution to systems, (sections 4,5,6,8 & 9 of chapter 5, chapter 6 (sections 1,3,5,6) of Text book 1).

Unit-IV

Integral equations, Differentiation of a function under an integral sign, Relation between differential and integral equation, Solution of non-homogeneous Volterra’s integral equation of second kind by the methods of successive substitution and successive approximation. (Chapter 1 & chapter 2 (sections 2.1, 2.2) of Text book 2).

Unit-V

Determination of some resolvent kernels, Volterra integral equation of first kind, Solution of the Fredholm integral equation by the method of successive substitutions, Iterated kernels, Solution of the Fredholm’s integral equation by the methods of successive substitution and successive approximation, Reciprocal functions, Volterra’s solution of Fredholm’s equation. (Chapter 2 (excluding 2.1, 2.2)of Text book 2).

Text books:

1. An Introduction to ordinary differential equations by E.A. Coddington, Prentice Hall of India Pvt. Ltd., New Delhi, 1987.

2. Integral equations by Shanti Swarup, Krishna Prakashan Media (P) Ltd, Meerut, 2003.

M.Sc Degree Examination

First Semester

Applied Mathematics

21AM102: Ordinary Differential Equations& Integral Equations

(Effective from the admitted batch of 2021-2022)

Time: 3 hours Maximum: 80 marks

Answer one question from each unit. All questions carry equal marks

UNIT-I

1. a) Let $\phi_1, \phi_2,...,\phi_n$ be n linearly independent solutions of L(y)=$y^{(n)} + a_1(x)y^{(n-1)} + \cdots + a_n(x)y$= 0 , on an interval I. Show that any solution φ of L(y)=0 on I, is of the form $\varphi = c_1\varphi_1 + \cdots + c_n\varphi_n$ where $c_1, \ldots, c_n$ are constants.

   b) One solution of $x^3y''' - 3x^2y'' + 6xy' - 6y = 0, \forall\, x > 0$ is $\phi_1(x) = x$. Find the basis of the above differential equation $x > 0$.

OR

2. a) Let $\phi_1, \phi_2,...,\phi_n$ be n solutions of L(y)=0 on an interval I, prove that they are linearly independent if and only if $W(\phi_1, \phi_2.....\phi_n)(x) \neq 0$ for all x in I.

   b) Two solutions of $x^3y''' - 3\,xy' + 3y = 0\ (x > 0\ )$ are $\phi_1(x) = x$, $\phi_2(x) = x^3$, then find a third independent solution.

UNIT-II

3. a) Consider the equation $x^2y'' + xe^xy' + y = 0$

   i)Compute the indicial polynomial; and show that its roots are –i and i.

   ii)Compute the coefficients C_1, C_2, C_3 in the solution

   $$\phi(x) = x^i \sum_{k=0}^{\infty} C_k x^k (x > 0)\ ,\ C_0 = 1.$$

   b) i)Show that -1 and 1 are regular singular points for the Legendre equation $(1 - x^2)y'' - 2xyy' + \alpha(\alpha + 1)y = 0$.

   ii) Find the indicial polynomial and its roots, corresponding to the point $x = 1$.

OR

4. a) Let M,N be two real valued functions which have continuous first partial derivatives on some rectangle $R: |x - x_0| \leq a,\ |y - y_0| \leq b$. Then

prove that the equation $M(x,y) + N(x,y)y' = 0$ is exact in R if and only if $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$.

b) Find an integrating factor for the following equation $(2y^3 + 2)dx + 3xy^2\, dy = 0$ and solve it.

UNIT-III

5. a) State and prove Picards existence theorem on successive approximation for the solution of I.V.P

OR

6. a) Find a solution φ of the system

$$y_1' = y_2,$$
$$y_2' = 6y_1 + y_2, \text{ satisfying } \varphi(0) = (1, -1)\ .$$

b) Show that the function f given by $f(x,y) = x^2|y|$ satisfies a Lipschitz condition on $R: |x| \le 1,\ |y| \le 1$, and find Lipschitz constant.

UNIT-IV

7. a) Obtain Fredholm integral equation of second kind corresponding to the boundary value problem $\frac{d^2\phi}{dx^2} + x\phi = 1,\ \ \phi(0) = 0, \phi(1) = 1$, also recover the boundary value problem from the obtained integral equation.

b) Solve the integral equation $\varphi(x) = (1 + x) + \int_0^x (x - s)\varphi(s)ds$ with $\varphi_0(x) = 1$, using the method of successive approximations.

OR

8. a) Convert the differential equation $\frac{d^2\phi}{dx^2} - 2x\frac{d\phi}{dx} - 3\phi = 0$ with the initial conditions $\phi(0) = 0, \phi'(0) = 0$ to Volterra's integral equation of second kind, conversely derive the original differential equation with the initial conditions from the integral equation obtained.

b) Find the resolvent kernel of the Volterra's integral equation with the kernel $k(x,\xi) = 1$.

UNIT-V

9. a) Find the resolvent kernel of the Volterra's integral equation with the kernel $k(x,u) = \frac{2+cosx}{2+cosu}$ and there by solve the integral equation

$$\phi(x) = e^x sinx + \int_0^x \frac{2+cosx}{2+cosu}\ \phi(u)\, du.$$

b) Find the solution of the integral equation $\varphi(x) = 1 + x^2 + \int_0^x \frac{1+x^2}{1+s^2}\varphi(s)ds$ with the help of the resolvent kernel.

OR

10. a) Find the iterated kernel for $k(x,\xi) = x - \xi$ if $a = 0, b = 1$.
b) Solve the following integral equation $\varphi(x) = \frac{5x}{6} + \frac{1}{2}\int_0^1 x\xi\varphi(\xi)d\xi$.

M.Sc. APPLIED MATHEMATICS (SECOND SEMESTER)

21AM 103: CLASSICAL MECHANICS

(With effect from 2021-2022 Admitted Batch)

Course Code:21AM103

(A total of ten questions are to be set as internal choice in each unit and the student has to answer one question from each unit. Each question carries 16 marks.)

Course Outcomes(COs): At the end of the course, the students will be able to

CO1: To know what central, conservative and central-conservative forces, mathematically understand the conservative theorems of energy, linear momentum and angular momentum. To know the importance of concepts such as generalized coordinates and constrained motion.

CO2: To know how to impose constraints on a system in order to simplify the methods to be used in solving physics problems in Lagrangian mechanics. To know how to deduce Hamilton's equations from variational principle.

CO3: Understand the concepts of canonical transformations, Poisson and Lagrange brackets.

CO4: To find the linear approximation to any dynamical system near equilibrium and also know how to derive and solve the equations of motion for the dynamical system using Hamilton-Jacobi method.

CO5: To distinguish between 'inertia frame of reference' and 'non-inertial frame of reference'. Also know about Lorentz transformations and consequences of Lorentz transformations.

fundamental problems.

Course Specific Outcomes(CSOs):

CSO1: This course able to develop basic mechanical concepts related to discrete and continuous mechanical systems.

CSO2: Connect concepts and mathematical rigor in order to enhance understanding.

CSO3: Describe and understand the motion of a mechanical system using Lagrange Hamilton formalism.

Learning Outcomes: Upon successful completion of this course, it is intended that a student will be able to

LO1: learn about Lagrangian and Hamiltonian formulation of Classical Mechanics.

LO2: State the conservation principles involving momentum, angular momentum and energy and understand that they follow from the fundamental equations of motion.

LO3: Have a deep understanding of canonical transformations.

LO4: learn about motion of a particle under various constraints.

LO5: Use Euler-Lagrange equation to find stationary paths and its applications in some classical

Mapping of course outcomes with the program outcomes					
	PO1	**PO2**	**PO3**	**PO4**	**PO5**
CO1	**√**	-	**√**	**√**	-
CO2	**√**	-	**√**	**√**	-
CO3	**√**	**√**	**√**	**√**	-
CO4	**√**	-	**√**	**√**	-
CO5	**√**	-	**√**	**√**	-

Unit-I

Lagrangian Formulation: Mechanics of a particle, mechanics of a system of particles, constraints, generalized coordinates generalized velocity, generalized force and potential. D'Alembert's principle and Lagranges equations, some applications of Lagrangian formulation (scope and treatment as in Art.1.1 to 1.4 and Art 1.6 of Text book.1).

Unit-II

Hamilton's principle, derivation of Lagrange's equations from Hamilton's principle, extension of Hamilton's principle to non-holonomic systems, advantages of variational principle formulation, conservation theorems and symmetry properties (scope and treatment as in Art 2.1 and 2.3 to 2.6 of Text book.1).

Unit-III

Hamiltonian formulation: Legendre transformations and the Hamilton equations of motion, cyclic coordinates and conservation theorems, derivation of Hamilton's equations from a variational principle, the principle of least action, the equation of canonical transformation, examples of canonical transformation, the Harmonic Oscillator, the symplectic approach to canonical transformations (scope and treatment as in Art.8.1,8.2,8.5, 8.6 and 9.1 to 9.4 of Text book.1).

Unit-IV

Poisson and Lagrange brackets and their invariance under canonical transformation. Jacobi's identity; Poisson's Theorem. Equations of motion infinitesimal canonical transformation in the Poisson bracket formulation. Hamilton Jacobi Equations for Hamilton's principal function, The harmonic oscillator problem as an example of the Hamilton – Jacobi method, the Hamilton – Jacobi equation for Hamilton's characteristic function (scope and treatment as in Art 9.5,9.6, 10.1, 10.2 and 10.3 of Text book.1)

Unit-V

New concept of space and Time, postulates of special theory of relativity, Lorentz transformation equations, Lorentz contraction, Time dilation, simultaneity, Relativistic formulae for composition of velocities and accelerations, proper time, Lorentz transformations form a group (scope and treatment as in chapters 1 and 2 of Text book.2).

Text books:

1. Classical mechanics by H.Goldstein, 2nd edition, Narosa Publishing House.
2. Relevant topics from Special relativity by W.Rindler, Oliver & Boyd, 1960.

M.Sc. Degree Examination

First Semester

Applied Mathematics

21AM103:Classical Mechanics

(Effective from admitted batch of 2021-2022)

Time: Three hours Maximum: 80 marks

Answer one question from each unit. All questions carry equal marks

Unit-I

1). (a) State and explain conservation principle of angular momentum for a single particle.

(b) State and obtain Nielsen's form of the Lagrange's equations for aholonomic dynamical system.

(OR)

2). (a)State and explain D' Alembert's principle.

(b)Derive the Lagrange's equations of motion from the D' Alembert's principle.

Unit-II

3).(a)Derive the Hamilton's principle from the D' Alembert's principle.

(b)What is cyclic or ignorable coordinate. Prove that the generalized momentum conjugate to a cyclic coordinate is conserved.

(OR)

4). (a)Derive Lagrange's equations of motion from Hamilton's principle.

(b) Determine the acceleration of the two masses of a simple Atwood machine, with one fixed pulley and two masses m_1 and m_2.

Unit-III

5). (a) Derive the Hamilton's equations of motion from a variational principle.

(b) Obtain Hamilton's Canonical equations of motion for a simple pendulum.

(OR)

6). (a) State and proveprincipleof least action.

(b) Discuss harmonic oscillator as an example of canonical transformations.

Unit-IV

7). (a) State and prove Jacobi's Identity.

(b)Prove the invariance of Poisson brackets with respect to canonical transformation.

(OR)

8). (a) For what values of α and β do the equations $Q = q^{\alpha} cos(\beta p),\ P = q^{\alpha} sin(\beta p)$ represent a canonical transformation?

(b) Find the motion of one dimensional simple harmonic oscillator by Hamilton- Jacobi method.

Unit-V

9). Derive Lorentz transformation equations.

(OR)

10). (a)Explain the following:

(i) Longitudinal contraction effect.

(ii) Simultaneity.

(iii) Proper time.

(b) Show that $ds^2 = -(dx)^2 - (dy)^2 - (dz)^2 + c^2(dt)^2$is invariant under Lorentz transformation.

M.Sc. APPLIED MATHEMATICS (FIRST SEMESTER)

21AM104: DISCRETE MATHEMATICAL STRUCTURES
(With effect from 2021-2022 Admitted Batch)
Course Code:21AM104

(A total of five questions are to be set as internal choice with one question from each unit and each question carries 16 marks.)

Course Outcome(COs): At the end of the course, the students will be able to

CO1: familiar to mathematical logic in discrete mathematics and various results.
CO2: Theory of inference and predicate calculus are taught.
CO3:familiarize the students with relations and lattice theory.
CO4: Basic concepts as well as different types of graphs in graph theory are taught.
CO5: understand and use algorithms in graph theory.

Course specific outcome(CSOs):

CSO1:To introduce the students to the topics and techniques of discrete methods and combinatorial reasoning.
CSO2: To introduce a wide variety of applications. The algorithmic approach to the solution of problems is fundamental in discrete mathematics, and this approach reinforces the close ties between this discipline and the area of computer science.
CSO3: To familiarize the students to graph theory and its applications.

Learning outcomes(LOs):

LO1:Student will be able to demonstrate skills in solving mathematical problems
LO2: Student will be able to comprehend mathematical principles and logic
LO3: Student will be able to demonstrate knowledge of mathematical modeling and proficiency in using mathematical software
LO4: Student will be able to manipulate and analyze data numerically and/or graphically using appropriate Software
LO5: Student will be able to communicate effectively mathematical ideas/results verbally or in writing.

Mapping of course outcomes with the program outcomes:

	PO1	PO2	PO3	PO4	PO5
CO1	✓	✓	✓	✓	✓
CO2	✓	✓		✓	✓
CO3	✓		✓	✓	✓
CO4	✓	✓	✓		✓
CO5	✓	✓	✓	✓	✓

Unit-I

Mathematical logic: statements structures and notation, connectives, well formed formulas, tautologies, equivalences, implications, normal forms – Disjunctive and conjunctive, Principle disjunctive and conjunctive normal forms. (Scope and treatment as in Sections: 1.1 to 1.3 of Text book 1)

Unit-II

Theory of Inference: Theory of inferences for statement calculus, validity using truth tables, rules of Inference. Predicate calculus: predicates, predicate formulas, quantifiers, free and bound variables, Inference theory of predicate calculus. (Scope and treatment as in Sections: 1.4 to 1.6 of Text book 1)

Unit-III

Relations and ordering: partially ordered relations, Partially ordered sets, representation and associated terminology. (Sections 2-3.1, 2-3.2, 2-3.8, 2-3.9 of Chapter 2 in Text book1)
Lattices, Lattices as partially ordered sets, some properties of Lattices, Lattices as algebraic systems, sub-Lattices, direct product and homomorphism some special Lattices.
(Sections: 4-1.1 to 4-1.5 of chapter 4 of Text book.1).

Unit-IV

Graph Theory: Graphs and Multigraphs, Subgraphs, Isomorphism and Homomorphism, Paths, Connectivity, Traversable Multigraph, Labeled and Weighted Graphs, Complete, Regular and Bipartite Graphs, Trees, Planar Graphs. (Scope as in Sections 8.2 to 8.9 of chapter 8 of textbook 2).

Unit-V

Directed Graphs: Rooted Trees, Sequential Representation of Directed Graphs,Warshall's Algorithm, Shortest Path, Binary Trees, Complete and Extended Binary Trees, Representation of Binary Trees, Traversing Binary Trees and Binary Search Trees (Scope as in Sections 9.2 to 9.6 and 9.8 of chapter 9 and 10.1 to 10.6 of chapter 10 of textbook 2).

Text books:

1. Discrete Mathematical structures with Applications to Computer Science by J.P. Trembly and R.Manohar, Tata Mc.Graw hill.

2. Discrete Mathematics, Schaum's outline series, second edition, by Seymour Lipschutz and Marc Lipson Tata Mc Graw-Hill.

M.Sc Degree Examination

First Semester

Applied Mathematics

21AM104: Discrete Mathematical Structures

(Effective from the admitted batch of 2021-2022)

Time: 3 hours Maximum: 80 marks

Answer one question from each unit. All questions carry equal marks

UNIT-I

1. a) Show the following implications without constructing the truth tables.
   $(P \to Q) \to Q \Rightarrow P \vee Q$.
   b) Show that the following are equivalent formulas.
   i) $P \vee (P \wedge Q) \Leftrightarrow P$.
   ii) $(P \vee \urcorner P \wedge Q) \Leftrightarrow P \vee Q$.

(or)

2. a) Obtain the principal disjunctive normal form of $P \to ((P \to Q) \wedge \urcorner(\urcorner Q \vee \urcorner P))$
   b) Obtain the principal conjunctive normal form of the formula $(\urcorner P \to R) \wedge (Q \leftrightarrows P)$.

UNIT-II

3. a) Demonstrate that R is a valid inference from the premises $P \to Q, Q \to R$, and P.
   b) Show that $S \vee R$ is tautologically implied by $(P \vee Q) \wedge (P \to R) \wedge (Q \to S)$.

(or)

4. a) Show that the following premises are inconsistent, $P \to Q, P \to R, Q \to \urcorner R$, P.
   b) Show that $(x)(P(x) \vee Q(x)) \Rightarrow (x)P(x) \vee (\exists x)Q(x)$.

UNIT-III

5. a) If R is a partial ordering relation on a set X and $A \subseteq X$, Show that $R \cap (A \times A)$ is a partial ordering relation on A.
   b) Let A be a given finite set and $\rho(A)$ its power set. Let $\subseteq$ be the inclusion relation on the elements of $\rho(A)$. Draw Hasse diagrams of $< \rho(A), \subseteq >$ for
   i) A={a} ii) A={a,b} iii) A={a,b,c} iv) A={a,b,c,d}

(or)

6. a) Let <L,≤> be a lattice in which $*$ and $\oplus$ denote the operation of meet and join respectively. Prove that for any a, b,∈ $L, a \leq b \Leftrightarrow a * b = a \Leftrightarrow a \oplus b = b$.
   b) Prove that every chain is a distributive lattice.

UNIT-IV

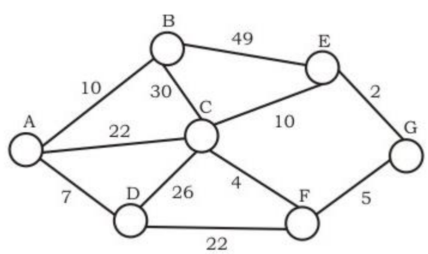
7. a) State and derive Euler's formula for graphs.
   b) Show that the number of vertices of odd degree in any graph is always even.

(or)

8. a) Define Hamiltonian and Eulerian graphs and give examples. Also give an example of a graph which is Eulerian but not Hamiltonian.
   b) Prove that a tree with n vertices has exactly (n-1) edges.

UNIT-V

9. a) Write Warshall's algorithm to find the shortest path in graphs.
   b) Find the minimal spanning tree of the following graph G and find the total weight of the minimal spanning tree by using Prim's algorithm.



(or)

10. a) Write Depth-First Search algorithm to find the spanning tree.
    b) Define a binary tree and draw the binary tree T which corresponds to the algebraic expression E=$(x+3y)^4(a-2b)$.

M.Sc. APPLIED MATHEMATICS (FIRST SEMESTER)

21AM 105: PROGRAMMING IN C

(With effect from 2021-2022 Admitted Batch)

Course Code:21AM105

(A total of Ten questions to be set by selecting two questions from each unit with internal choice. Each question carries 16 marks.)

Course Outcome(COs):

CO1: All the syntax rules of the programming language C are taught.

CO2: The syntax rules may applied to develop programs for various general problems and mathematical problems.

CO3: The advanced concepts of storing group of homogeneous elements in one name for numerical values and characters.

C04: Handling memory locations and storing heterogeneous group of elements in a common name .

CO5: Scope and extent of variables are taught and using these concepts programs are to be developed.

Course Specific Outcomes(CSOs):

CSO1: Able to develop logic to solve problems and preparing flowchart/algorithm for the solution.

CSO2: Able to apply syntax rules of the language to develop programs.

CSO3: Able to use advanced level techniques like representing group of values with single name, sharing memory and frequently used part as a subprograms.

Learning outcomes(LOs):

LO1: Students may familiar with different syntax rules of the programming language C.

LO2:Students are able to think logically in solving problems.

LO3: Able to develop program on various concepts using control statements.

LO4: Able to develop programs using one dimensional, two dimensional and character arrays.

LO5: Students may familiar to use functions, pointers and structures in developing programs.

Mapping of course outcomes with the program outcomes

	PO1	PO2	PO3	PO4	PO5
CO1	✓	✓			✓
CO2	✓	✓			✓
CO3	✓	✓	✓		
CO4	✓		✓		✓
CO5	✓	✓	✓		✓

Unit-I

Data types, Operators and Some statements: Identifiers and key words, Constants, C operators, Type conversion. Writing a Program in C: Variable declaration, Statements, Simple C Programs, Simple input statement, Simple output statement, Featutre of stdio.h.

Control statements: Conditional expressions: If statement, if-else statement.

Unit-II

Switch statement, Loop statements: For loop, While loop, Do – while loop, Breaking control statements: Break statement, Continue statement, goto statement.

Unit-III

Functions and Program Structures: Introduction, Defining a function, Return statement, Types of Functions, Actual and formal arguments, Local Global variables. The scope of variables: Automatic Variables, Register Variables, Static Variables, External variables, Recursive functions.

Unit-IV

Arrays: Array Notation, Array declaration, Array initialization, Processing with arrays, Arrays and functions, Multidimensional array, Character array.

Pointers: Pointer declaration, Pointer operator, Address Operator, Pointer expressions, Pointer arithmetic.

Unit-V

Pointers: Pointers and functions, Call by value, Call by reference, Pointers and arrays, Pointer and one dimensional array, Pointer and multidimensional array, Pointer and strings, Array of pointers, Pointers to pointers. Structures, Unions: Declaration of Structure, Initializing a structure, Functions and Structures, Arrays of Structures, Arrays within a structure, Structure within a structure, Pointers and structures, Unions.

Text books:

Programming in C by D.Ravichandran, New Age International, 1998 Chapters: 1, 2, 3,4,5,6, and 8.

Reference: Programming in C by E.Balaguruswamy, 6th Edition, McGraw Hill Education.

M.Sc Degree Examination
First Semester
Applied Mathematics
21AM105:Programming in C
(Effective from the admitted batch of 2021-2022)

Time: Three hours Maximum: 80 marks

Answer one question from each unit. All questions carry equal marks

Unit-I

1. (a) Discuss about operators available in C language.
   (b) Write a program in C to perform the following
   (i) Area of a circle, (ii) Circumference of a circle,
   (iii) Area of a triangle, (iv) Area of a rectangle.

(or)

2.(a)Write and explain the general forms of nested if statements.
   (b) Write a program in C to find the roots of quadratic equation using if else structure.

Unit-II

3. (a) Explain about various loop statements.
   (b) Write a C programming to check give number is palindrome or not.

(or)

4.(a) Write a program to generate prime numbers in the given range.
   (b) Write a program in C to convert given decimal number to octal number.

Unit-III

5. (a)Write a general form of the function and also write three types of functions.
   (b) Write a function to swap the values of two variables, and corresponding main program

(or)

6.(a) Explain about four different types of storage classes available in C.
   (b) Write a recursive function to compute factorial of a given integer.

Unit-IV

7.(a)Write a function to compute norm of a matrix.
   (b) Write a program in C to compute transpose of a matrix.

(or)

8.(c) Explain the following (i) Pointer variable, (ii) Pointer operator, (iii) Address operator

(d) Write a program to copy a string to another string.

Unit-V

9. (a) Explain about call by value and call by reference and give examples.
   (b) Write a program tosort set of n numbers in ascending order using pointers.

(or)

10 (a) Explain the relation between
   (i) pointer and one dimensional array, (ii) pointers and multi dimensional arrays.
   (b) Write the general form of a structure and create a structure for students data with roll no, age, sex, height and weight and write a program to read and print the contents of the structure.

Lab: Programming in C language

Code: 21AMPR101

Course Outcome:

CO1: Development of programs for computations various problems.

CO2: Editing ,compiling and debuging of the programs.

CO3: Extending the programs with certain modifications.

CO4: Development of programs to compute numerical solutions of differential equations.

CO5: Using pointers and functions developing programs.

Learning Outcome:

LO1: Able to develop and run the programs using control statements

LO2: Able to develop and run programs using loop statements.

LO3: Able to develop programs for group of homogeneous values.

LO4: Able to develop programs using functions.

LO5: Able to develop programs using pointers.

1. Program to solve quadratic equation using switch case structure.
2. Program to convert a given decimal number to octal number.
3. Program to generate prime numbers in a given range.
4. Program to check a given integer is a palindrome.
5. Sorting of numbers using arrays.
6. Compute multiplication of matrices.
7. Compute norm of a matrix using functions
8. Computing numerical integration using Simpson and Tripazodal rules
9. Solving ODE with initial conditions using Adams Bashforth method
10. Solving ODE by fourth Runge-Kutta method.
11. Compute binomial coefficients using recursive function for factorial.
12. Program to check a given string is palindrome .
13. Using pointers copy a string to another string.
14. Using pointers and functions sorting numbers.

M.Sc. APPLIED MATHEMATICS (SECOND SEMESTER)
21AM 201: COMPLEX ANALYSIS
(With effect from 2021-2022 Admitted Batch)
Course Code:21AM201

(A total of ten questions are to be set as internal choice in each unit and the student has to answer one question from each unit. Each question carries 16 marks.)

Course Outcomes: At the end of the course, the students will be able to

CO1: Represent complex numbers algebraically and geometrically. Define and analyze limits and continuity for complex functions as well as consequences of continuity.

CO2: Apply the concept and consequences of analyticity and the Cauchy-Riemann equations and of results on harmonic and entire functions including the fundamental theorem of algebra.

CO3: Evaluate complex contour integrals directly and by the fundamental theorem, apply the Cauchy-Goursat theorem in its extended versions, and the Cauchy integral formula.

CO4: Represent functions as Taylor, power and Laurent series, classify singularities and poles, find residues and evaluate complex integrals using the residue theorem.

CO5: Describe conformal mapping properties of elementary functions and mapping properties of some special transcendental functions.

Course Specific Outcomes:

CSO1: Understanding of the fundamental axioms in complex analysis and capability of developing ideas based on them.

CSO2: Prepare and motivate students for research studies in mathematical analysis and related fields.

CSO3: Provide knowledge of a wide range of mathematical techniques and application of complex integration methods in other scientific and engineering domains.

Learning Outcomes: Upon successful completion of this course, it is intended that a student will be able to

LO1: Explain the fundamental concepts of complex analysis and their role in modern mathematics and applied contexts.

LO2: Demonstrate accurate and efficient use of complex analysis techniques.

LO3: Demonstrate capacity for mathematical reasoning through analysing, proving and explaining concepts from complex analysis.

LO4: Apply contour integration techniques to diverse situations in physics, engineering and other mathematical contexts.

LO5: Understand relations between conformal mappings and quadratic differentials and how geometric structures are changing under conformal mappings.

Mapping of course outcomes with the program outcomes					
	PO1	**PO2**	**PO3**	**PO4**	**PO5**
CO1	√	-	√	√	-
CO2	√	-	√	√	-
CO3	√	-	√	√	-
CO4	√	√	√	√	-
CO5	√	√	√	√	-

Unit-I

Analytic and Harmonic functions: Differentiable and analytic functions, Cauchy – Riemann equations, Harmonic functions. Elementary functions: The complex exponential function, The complex logarithm function, Complex exponents, Trigonometric and hyperbolic functions, Inverse trigonometric and hyperbolic functions.

Unit-II

Complex integration: Complex integrals, Contours and contour integrals, The Cauchy – Goursat theorem, The fundamental theorems of integration, Integral representations for analytic functions, The theorems of Morera and Liouville and extensions.

Unit-III

Taylor and Laurent series: Uniform convergence, Taylor series representations, Laurent series representations, singularities, Zeros and poles, Applications of Taylor and Laurent series.

Unit-IV

Residue theory: The residue theorem, Trigonometric integrals, Improper integrals of rational functions, Improper integrals involving trigonometric functions, Indented contour integrals, Integrals with branch point, The Argument principle and Rouche's theorem.

Unit-V

Conformal mapping: Basic properties of conformal mapping, Bilinear transformations, Mappings involving elementary functions, Mapping by trigonometric functions.

Text book: Complex analysis for Mathematics and Engineering–5th Edition by John H. Mathews and Russel W, Howell. Narosa publishing house **(Chapters: 3, 5,6, 7, 8 & 10)**

M.Sc Degree Examination
Second Semester
Applied Mathematics
21AM201: Complex Analysis
(Effective from the admitted batch of 2021-2022)

Time: 3 hours Maximum: 80 marks

Answer one question from each unit. All questions carry equal marks

UNIT – I

1). (a) Let $f(z) = f(x + iy) = u(x, y) + iv(x, y)$be differentiable at the point $z_0 = x_0 + y_0$.Then prove that the partial derivatives of u and v exist at the point(x_0, y_0) and satisfy the equations $u_x(x_0, y_0) = v_y(x_0, y_0)$ and$u_y(x_0, y_0) = -v_x(x_0, y_0)$.

(b) Let f be an analytic function in the domain D. If $|f(z)|$=k, where k is a constant, then prove that f is constantin D.

(OR)

2). (a) Let $f(z) = u(x, y) + iv(x, y)$ be an analytic function in the domain D. If all second order partial derivatives of u and v are continuous then prove that u and v are harmonic functions in D.

(b) Show that $u(x,y) = xy^3 - x^3y$ is a harmonic function and find the conjugate harmonic function $v(x,y)$.

UNIT – II

3).(a) State and prove Cauchy- Goursat Theorem.

(b) State and prove Liouville's theorem.

(OR)

4) (a) State and prove Cauchy's integral formula.

(b) Evaluate the following integral $\int_c (z - 3/z^2 + 2z + 5)dz$,where C is the circle $|z| = 1$.

UNIT –III

5) State and prove Laurent's theorem.

(OR)

6) (a) Obtain two Laurent series expansion in powers of $z$for the function $f(z) = \frac{1}{z^2(1-z)}$ and specify the region in which those expansions are valid.

(b) Locate the singularities of the following functions and determine their type:

(i) z exp($\frac{1}{z}$) (ii) $\frac{z^2}{z-sinz}$

UNIT-IV

7). (a) State and prove Cauchy's residue theorem.

(b) Using the theory of residues, evaluate $\int_0^\infty \frac{x^2}{(x^2+1)(x^2+4)} dx$

(OR)

8). (a) State and prove Rouche's theorem.

(b) Let $g(z) = z^5 + 4z - 15$

(i). Show that there are no zeros in $|z| < 1$.

(ii). Show that there are five zeros in $|z| < 2$.

UNIT -V

9). (a) Find the image of the upper half plane $Im(z) > 0$ under the transformation $w = \frac{(1-i)z+2}{(1+i)z+2}$.

(b) Find the bilinear transformation $w = s(z)$ that maps the points $z_1 = 0, z_2 = i\ and\ z_3 = -i\ onto\ w_1 = -1, w_2 = 1\ and\ w_3 = 0$ respectively.

(OR)

10). (a) Find the fixed points of

(i). $w = \frac{z-1}{z+1}$(ii). $w = \frac{4z+3}{2z-1}$

(b) Show that $\boldsymbol{w} = \boldsymbol{s}(\boldsymbol{z}) = \frac{\boldsymbol{i}(\mathbf{1}-\boldsymbol{z})}{\mathbf{1}+\boldsymbol{z}}$ maps the unit disk $|\boldsymbol{z}| < \mathbf{1}$ one-to-one and onto the upper half plane $Im(z) > 0$.

M.Sc. APPLIED MATHEMATICS (SECOND SEMESTER)

21AM 202: PARTIAL DIFFERENTIAL EQUATIONS & INTEGRAL TRANSFORMS

(With effect from 2021-2022 Admitted Batch)

Course Code:21AM202

(A total of Ten questions to be set by selecting two questions from each unit with internal choice. Each question carries 16 marks.)

Course Outcome(COs):

CO1: Familiar with formulation and solving first order pdes by various methods

CO2:Familiar with classifying, reducing to canonical forms and solving second order pdes.

CO3: Familiar with modelling the vibrating string, wave equation and their solutions its applications by second order pdes.

CO4: Familiar with Laplace transforms, their properties and solving equations using with the transforms.

CO5: Familiar with Fourier transforms, their properties and its applications.

Course Specific Outcome(CSOs):

CSO1: First order and second order partial differential equations their classifications and solving them.

CSO2: Modelling the physical systems by second order partial differential equations and their interpretation.

CSO3: Integral Transforms their validity and its applications to solving various problems.

Learning Outcomes(LOs):

LO1: Able to formulate and solving partial differential equations of first order.

LO2: Able to classify, reducing and solving higher order partial differential equations.

LO3: Able to formulate mathematical models for certain physical systems and computing the solutions.

LO4: Able to understand the existence, definition and properties of Laplace transforms and its applications.

LO5: Able to understand the concept of Fourier transforms and solving initial and boundary value problems.

Mapping of course outcomes with the program outcomes:

	PO1	PO2	PO3	PO4	PO5
CO1	✓	✓	✓	✓	
CO2	✓	✓	✓	✓	
CO3	✓	✓		✓	
CO4	✓	✓	✓	✓	
CO5	✓		✓	✓	

Unit-I

Partial differential equations: Equations of the form dx/p=dy/q=dz/r, Orthogonal trajectories, Pfaffian differential equations, 1st order partial differential equations; Charpit's method and some special methods. Jacobi's method. (Chapter 1 (excluding sections 7 & 8), Chapter-II (excluding section 14) of Text book 1).

Unit-II

Second order Partial differential equations with constant & variable coefficients, canonical forms, separation of variables method, Monge's method (Chapter III (excluding section 10) of Text book 1).

Unit-III

Partial differential equations – Modeling : vibrating string, one-dimensional wave equation, separation of variables, D'Alembert's solution of the wave equation, one-dimensional heat flow, heat flow in an infinite bar, two-dimensional wave equation. (Chapter 11 – Section 11.2 to 11.7 of Text book 2)

Unit-IV

Laplace Transform definition, conditions for existence, properties, problems, inverse Laplace transforms, convolution theorem, applications of convolution theorems, solutions of ordinary, partial differential equations using Laplace transforms.(Chapters 1,2,3 of Text book 3)

Unit-V

Fourier Transform definition, conditions for existence, properties, problems, inverse Fourier transforms, relation between Laplace and Fourier Transforms, Fourier sine transforms, Fourier Cosine transform, finite Fourier transforms, applications of convolution theorems, solutions of ordinary, partial differential equations using Fourier transforms (Chapters 6 and 8 (sections 8.1 & 8.2 only) of Text book 3)

Text books:

1. I.N. Sneddon, Elements of partial differential equations. Mc Graw Hill International student Edition, 1964.
2. Advanced Engineering Mathematics by Erwin Kreyszig, 5th Edition, New Age International (P) Limited Publishers.
3. A.R.Vasishtha & R.K.Gupta, Integral transforms, Krishna Prakashan Media (P) Ltd, Meerut, 2003.

M.Sc Degree Examination

Second Semester

Applied Mathematics

21AM202: Partial Differential Equations & Integral Transforms

(Effective from the admitted batch of 2021-2022)

Time: 3 hours　　　　Maximum: 80 marks

Answer one question from each unit. All questions carry equal marks

UNIT-1

1. (a) Find the integral curves for the sets of equations

$$\frac{dx}{xz-y} = \frac{dy}{yz-x} = \frac{dz}{1-z^2}.$$

(b) Find the orthogonal trajectories on the cone $x^2 + y^2 = z^2 tan^2\alpha$ of its intersections, with the family of planes parallel to $z = 0$.

(OR)

2. (a) Prove that the pfaffian differential equation $\bar{X}.d\bar{r} = 0$ is integrable if and only if $\bar{X}.curl\bar{X} = 0.$

(b) Find the complete integrals of the equation $(p^2 + q^2)y = qz.$

UNIT-II

3. (a) Solve the equation $\frac{\partial^4 z}{\partial x^4} + \frac{\partial^4 z}{\partial y^4} = 2\frac{\partial^4 z}{\partial x^2 \partial y^2}$.

(b) Reduce the equation $\frac{\partial^2 z}{\partial x^2} = x^2 \frac{\partial^2 z}{\partial y^2}$ to canonical form.

(OR)

4. (a) Solve the wave equation $r = t$ by Monge's method.

(b) Solve the equation $r + 4s + t + rt - s^2 = 2$.

UNIT-III

5. (a)A string is stretched and fastened to two points at a distance l apart. Motion is started by displacing the string in the form $y = a \sin \frac{n\pi}{l}$, from which it is released at time $t = 0$. Show that the displacement of any point at a distance x from one end at time t is given by $y(x,t) = a \sin \frac{\pi x}{l} \cos \frac{\pi ct}{l}$.

OR

6. (a) A homogeneous rod of conducting material of length 100cm has its ends kept at zero temperature and the temperature initially is $u(x,0) = x,\ 0 \le x \le 50$
$u(x,0) = 100 - x,\ 50 \le x \le 100$. Find the temperature $u(x,t)$ at any time.

UNIT-IV

7. (a) Solve $(D^3 - 3D^2 + 3D - 1)y = t^2 e^t, y(0) = 1, y'(0) = 0, y''(0) = -2$

by using Laplace transform.

(b) A particle moves along the line so that its displacement X, from a fixed point at any time t is given by $X''(t) + 4X'(t) + 5X(t) = 80 \sin 5t$. Find its displacement at any time $t > 0$, if at $t = 0$ the particle is at rest $X = 0$.

(OR)

8. (a) Using convolution theorem, show that $\int_0^t \sin u \cos(t-u) du = \frac{t}{2} \sin t$.

(b) Using Laplace transform solve $\frac{\partial^2 y}{\partial x^2} + y = t \cos 2t$.
with $y = 0, y' = 0$ when $t = 0$.

UNIT – V

9. (a) Find the Fourier transform of *f(x)*, if $f(x) = \begin{cases} \frac{1}{2\varepsilon}, & |x| \le \varepsilon \\ 0, & |x| > \varepsilon \end{cases}$.

(b) State and prove Fourier integral theorem.

(OR)

10. (a) Find the Fourier cosine transform of $f(x) = \frac{1}{1+x^2}$ and hence derive Fourier sine transform of $\emptyset(x) = \frac{x}{1+x^2}$.

(b) Solve the one- dimensional wave equation $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$, using the Fourier transform to determine the displacement $y(x, t)$ of an infinite string, given that the string is initially at rest.

M.Sc. APPLIED MATHEMATICS(SECOND SEMESTER)

AM 203: STATISTICS AND DISTRIBUTION THEORY

(With effect from 2021-2022 Admitted Batch)

Course Code:21AM203

(A total of ten questions are to be set taking two questions from each unit with internal choice in each unit. Each question carries 16 marks.)

Course Outcome(COs):

CO1: This course introduces some key concepts of probability, random variables and distributions. Provides an in-depth knowledge of understanding the statistical theory with real time examples in various social sciences.

CO2: Knowledge of basic understanding of various statistical processes and related problem solving skills are developed.

CO3: Improves the logical thinking ability of the students and applying the skills using various software packages like SPSS, SAS, etc.

CO4: Increases the subject knowledge that helps in pursuing higher studies as well as getting employment.

Course Specific Outcome(CSOs):

CSO1: Study on the basic concepts ofprobability and statisticaltheory, random variables and various distributions.

CSO2: Introducing the concepts of important probability distributions, correlation and regression analysis and testing of hypothesis.

CSO3: Gain knowledge of finding unknown distribution functions, their properties, problem solving skills.

Learning Outcome:

LO1: After studying the students are expected to learn some basic concepts of probability and statistics.

LO2: Applying the concepts to tests of hypothesis under various situations.

LO3: Knowledge of basic understanding of various statistical processes and related problem solving skills are developed.
LO4: Learns the concepts of correlation and regression analysis and curve fitting.
LO5: Gains knowledge and orientation for applying the knowledge for problem solving through various statistical packages.

Mapping of course outcomes with the program outcomes

	PO1	PO2	PO3	PO4	PO5
CO1	✓	✓			
CO2			✓		
CO3			✓		
CO4	✓			✓	✓
CO5		✓		✓	✓

Unit-I

Random variables, distribution functions, Mathematical expectation and Generating functions:

One and two dimensional random variables (Discrete and Continuous), Distribution functions, joint and conditional distribution functions, probability mass function, probability density function, Transformation of Random variables.
Mathematical expectation, Moments of a distribution function, moment generating functions, characteristic functions and their properties, Chebychev inequality, probability generating functions. (Chapter 5, Chapter 6 except section 6.7 and Chapter 7-Sections 7.1, 7.2, 7.3, 7.5 and 7.9)

Unit-II

Probability Distributions:

Discrete Distributions-Binomial, Poisson and geometric distributions and their properties with applications. (Sections 8.1-8.5 and 8.7 of Chapter8)
Continuous distributions – Gamma, Beta, Cauchy, Normal distributions and their properties with applications (Sections 9.1, 9.2, 9.5, 9.6,9.7 and 9.12 of chapter 9)

Unit-III

1. **Correlation and Regression:**

Correlation, Karl Pearson's coefficient of correlation, Calculation of correlation coefficient for Bivariate frequency distribution, Spearman's rank correlation coefficient. Linear regression- regression coefficients and their properties, angle between regression lines, standard error of estimate, curvilinear regression (Chapter 10 and Chapter 11)

Unit-IV

2. **Large Sample Theory:**Types of sampling, tests of significance, procedure for testing of hypothesis, tests of significance for large samples, sampling of attributes, sampling of variables (Chapter 14)

Unit-V

5. Exact Sampling Distributions:

Exact sampling distributions, χ^2, t, F distributions and their applications. (Chapter 15 up to 15.6.4 and Chapter 16 up to 16.6 except 16.4)

TEXT BOOK: Fundamentals of Mathematical Statistics-S.C.Gupta and V.K.Kapoor, 11 edition Sultan Chand and Sons, New Delhi

REFERENCE: An introduction to probability theory and mathematical statistics – V.K.Rohatgi Wiley Eastern Ltd, New Delhi

M.Sc Degree Examination

Second Semester

Applied Mathematics

21AM203: Statistics & Distribution Theory

(Effective from the admitted batch of 2021-2022)

Time: 3 hours Maximum: 80 marks

Answer one question from each unit. All questions carry equal marks

UNIT-I

1) a) The distribution function of a random variable X is given by

$$F(x) = \begin{cases} 1 - K(1+x)e^{-x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

Find the value of K and the corresponding density function of X.

b) Define mathematical expectation and explain the additive and multiplicative properties of expectation of two random variables.

OR

2) a)State and prove Chebychev's inequality.

b) Let (X, Y) be jointly distributed with pdf

$$f(x,y) = \begin{cases} 2, & 0 < x < y < 1 \\ 0, & otherwise \end{cases}$$

Find the marginal and conditional pdfs of $X$and Y. Also compute $P(Y \geq \frac{1}{2} | X = \frac{1}{2})$ and $P(X \geq \frac{1}{3} | Y = \frac{2}{3})$.

UNIT–II

3) a)Derive the moment generating function of binomial distribution and explain the additive property of binomial random variate.

b) Define gamma and beta distributions and establish the relationship, if any, between the corresponding random variables.

OR

4) a) Explain the uses of normal distribution. Also show that the mean deviation about the mean for normal distribution is $\frac{4}{5}\sigma$ approximately.

b) The number of customers arriving per hour at a certain automobile service facility is assumed to follow a Poisson distribution with mean $\lambda = 7$.

(i) Compute the probability that more than 10 customers will arrive in a 2-hour period.

(ii) What is the mean number of arrivals during a 2-hour period?

UNIT –III

5) a) Find the angle between the two regression lines and explain the cases when (i) the two variables are uncorrelated (ii) variables are perfectly correlated.

b)The table below shows the IQ of 7 fathers and their eldest sons:

IQ of father	91	97	102	103	105	110	114
IQ of son	102	94	105	115	113	99	116

Calculate the correlation coefficient between the IQ of father and son and comment briefly whether this value supports the theory that IQ is an inherited factor.

OR

6) a) From the following information, calculate the regression equations:
$\sum x = 30; \sum y = 40; \sum xy = 214; \sum x^2 = 220; \sum y^2 = 340; N = 5$. Also find the coefficient of correlation.

b) Derive rank correlation coefficient and find its limits.

UNIT –IV

7) a) Explain the steps involved in testing a hypothesis.

b) A machine puts 9 imperfect articles in a sample of 200. After the machine is overhauled, it puts out 4 imperfect articles in a batch of 100. Test at 5% level of significance whether the machine has been improved after overhauling.

OR

8) a) In a sample of 300 units of manufactured products, 65 units were found to be defective and in another sample of 200 units there were 35 defectives. Is there any significant difference in the proportion of defectives in the samples at 5% level of significance?

b) In a test given to two groups of students, the marks obtained are as follows:

First group	19	22	23	45	50	35	56	44	39
Second Group	27	55	34	24	33	44	26	40	27

Examine the significance of the difference between the arithmetic mean of the marks secured by the two groups of the students. Test at 5% level of significance.

UNIT –V

9) a) Derive Student's t-distribution and find its variance.

b)A random sample of 220 students in a college were asked to give opinion in terms of Yes or No about the winning of their college team in a tournament. The following data is collected:

	Class in College		
	I year	II year	III year
Yes	43	20	37
No	23	57	40

Test whether there is any association between opinion and class in college using 5% level of significance?

OR

10) a) If X_1 and X_2 are two independent χ^2 variates with n_1 and n_2 degrees of freedom respectively, then prove that $\frac{X_1}{X_2}$ is a $\beta_2\left(\frac{n_1}{2}, \frac{n_2}{2}\right)$ variate.

b) Explain F distribution and derive the relation between t and F distribution.

M.Sc. APPLIED MATHEMATICS (SECOND SEMESTER)
21AM 204: Elements of Elasticity and Fluid Dynamics
(With effect from 2021-2022 Admitted Batch)
Course Code:21AM204

(A total of five questions are to be set as internal choice with one question from each unit and each question carries 16 marks.)

Course Outcomes(COs):

CO1: Familiar with basic concepts on fluid motion and their physical properties.

CO2: Familiar with Euler's equation of motion, Bernoulli's equation and Kelvin's circulation theorem ,their solutions and properties.

CO3: Familiar with two dimensional fluid flow, classification of flows, Milne-Thomson circle theorem, and Blasius theorem.

CO4: Familiar with basic concepts on Analysis of strain and their properties.

CO5: Familiar with basic concepts on stress and their properties.

Course specific Outcomes(CSOs):

CSO1: Differentiate between fluids and solids their properties.

CSO2: Properties of various fluid flows and their classifications.

CSO3: The properties of strain and stress components will be familiar.

Learning Outcomes(LOs):

LO1: Able to understand the equation of continuity and general analysis of fluid motion.

LO2: Able to understand the equation of motion of a fluid , Bernoulli's equation and circulation theorem.

LO3: Able to understand the two dimensional fluid flows and their properties.

LO4: Able to understand the various deformations and equation of compatibility.

LO5: Able to understand the properties of the stress , Mohr's Diagram and certain examples of stress.

Mapping of course outcomes with the program outcomes:

	PO1	PO2	PO3	PO4	PO5
CO1	✓	✓		✓	
CO2	✓	✓	✓		
CO3	✓	✓	✓	✓	
CO4	✓		✓	✓	
CO5	✓	✓	✓	✓	

Unit-I

Kinematics of fluids, real and ideal fluids, velocity of fluid at a point, streamlines and path lines, velocity potential, velocity vector, local and particle rates of change, equation of continuity, Acceleration of fluid, conditions at a rigid boundary, General analysis of fluid motion (Chapter 2 of Text book 1).

Unit-II

Equation of motion of a fluid, pressure at a point in a fluid at rest and in a moving fluid, conditions at a boundary of two in viscid immiscible fluids, Euler's equations of motion, Bernoulli's equation. Discussion of the case of steady motion under conservative body forces, Vortex motion, Kelvin's circulation theorem. Some further aspects of vortex motion (Chapter 3(excluding sections 3.8 to 3.11) of Text book 1).

Unit-III

Some two - dimensional flows: Meaning of two - dimensional flow, use of cylindrical polar coordinates, the stream function, the complex potential for two – dimensional, irrotational, incompressible flow, complex potential for standard two – dimensional flows, some worked examples, two - dimensional image systems. The Milne- Thomson circle theorem, the theorem of Blasius (Chapter 5(excluding sections 5.10 to 5.12) of Text book 1).

Unit-IV

Analysis of strain: Deformation, affine deformation, infinitesimal affine deformation, geometrical interpretation of the components of strain, strain quadric of Cauchy, principal directions, invariants, general infinitesimal deformation, Examples of strain, equations of compatibility, finite deformations. (Chapter 1 of Text book 2)

Unit-V

Analysis of stress, body and surface forces, stress tensor, equations of equilibrium, transformation of coordinates, stress quadric of Cauchy, Mohr's diagram, examples of stress (Chapter 2 of Text book2)

Text books:

1. Text book of Fluid Dynamics by F.Chorlton, CBS publishers and distributors, New Delhi.
2. Mathematical theory of Elasticity, by I.S.SOKOLNIKOFF 2nd edition; Tata Mc Graw Hill-New Delhi

M.Sc Degree Examination

Second Semester

Applied Mathematics

21AM204: Elements of Elasticity & Fluid Dynamics

(Effective from the admitted batch of 2021-2022)

Time: 3 hours Maximum: 80 marks

Answer one question from each unit. All questions carry equal marks

UNIT-I

1. (a) Derive the equation of continuity for an incompressible fluid.

(b)Explain general analysis of fluid motion.

(OR)

2 (a) Discuss the acceleration of a fluid.

(b) Test whether the motion specified by $\bar{q} = \frac{k^2\,(x\bar{j} - y\bar{i})}{x^2+y^2}$ where k is a non zero is a possible form of motion for an incompressible fluid, if so find its vorticity vector?

UNIT-II

3. (a) Define the circulation. State and prove Kelvin's circulation theorem.

b) Write about pitot tube and venturi tube.

(OR)

4. (a) With usual notation derive the Bernoulli's equation for inviscid irrotational fluid motion.

(b) Derive Euler’s equation of motion in vector form and write its cartesian form.

UNIT-III

5. (a) Discuss the steady uniform flow past a fixed long infinite circular cylinder.

(b) Describe the irrotational motion of an incompressible liquid for which the complex potential is $i\kappa \log z$ □.

(or)

6.a) State and prove the Milne-Thomson circle theorem.

b) Using this theorem discuss the steady uniform flow past a stationary circular cylinder

UNIT-IV

7. (a) Define an affine transformation and show that an affine transformation carries straight line segments into straight line segments

(b) Explain about principle strains and invariants.

(or)

8. (a) Write about geometrical interpretation of strain components e_{23}.

(b) Derive the equations of compatibility in terms of strain components.

UNIT-V

9. (a) Define a stress tensor and show that it is symmetric.

(b) Derive equations of equilibrium.

(or)

10. (a) Explain in detail about Mohr's circle.

(b) Prove that the expression $T_{11}+T_{22}+T_{33}$ is invariant under an orthogonal transformation of coordinates.

M.Sc. APPLIED MATHEMATICS (SECOND SEMESTER)

21AM 205: NUMERICAL ANALYSIS
(With effect from 2021-2022 Admitted Batch)
Course Code:21AM205

(A total of Ten questions to be set by selecting two questions from each unit with internal choice. Each question carries 16 marks.)

Course outcomes (COs): At the end of the course, the students will be able to

CO1: Identity and analyze different types of errors encountered in numerical computing

CO2:Apply the knowledge of Numerical Mathematics to solve problems efficiently arising in science, engineering.

CO3: Utilize the tools of the Numerical Mathematics in order to formulate the real-world problems from the view point of numerical mathematics.

CO4: Design, analyze and implement of numerical methods for solving different types of problems, viz. initial and boundary value problems of ordinary differential equations etc.

CO5: Create, select and apply appropriate numerical techniques with the understanding of their limitations so that any possible modification in these techniques could be carried out in further research.

CO6: Identify the challenging problems in continuous mathematics (which are difficult to deal with analytically) and find their appropriate solutions accurately and efficiently.

Course Specific outcomes (CSOs):

CSO1: Students will learn numerical techniques to solve differential equations.

CSO2: students will gain understanding in the theoretical and practical aspects of the use of numerical methods.

CSO3 : They will be able to handle challenging problems in and find their appropriate solutions efficiently.

Learning outcomes (LOs):

LO1: In this course, students will learn well-known numerical techniques to solve differential equations, root finding problems.

LO2: The objective will be to train students to understand why the numerical methods work, what type of errors to expect, and when an application might lead to difficulties.

LO3: In particular, the students will become proficient in: Understanding the theoretical and practical aspects of the use of numerical methods.

LO4: Implementing numerical methods for a variety of multidisciplinary applications.

LO5: Establishing the limitations, advantages, and disadvantages of numerical methods.

Mapping of course outcomes with the program outcomes:

	PO1	PO2	PO3	PO4	PO5
CO1	✓	✓	✓	✓	✓
CO2	✓	✓		✓	✓
CO3	✓		✓	✓	✓
CO4	✓	✓	✓		✓
CO5	✓	✓	✓	✓	✓

Unit-I

Numerical techniques of solving transcendental and polynomial equations: Bisection methods, secant method, Newton-Raphson method, Chebyshev method, Rate of convergence. (Sec 2.1-2.5 of Textbook 1)

Unit-II

Numerical techniques of solving system of Linear Algebraic equations: Triangularization method, Gauss elimination method, Gauss-Jordan method, Iterative methods: Jacobi method, Gauss-Seidel method. Numerical techniques of determining the eigen values and eigen vectors of a matrix: Jacobi method, Power method. (Sec 3.1-3.7, 3.11 of Text book 1).

Unit-III

Approximation: Lagrange interpolation, Hermite interpolation, Spline interpolation, Least squares approximation. Methods based on undetermined coefficients – Gauss Legendre, Gauss-Chebyshev integration methods, Lobatto integration, Composite integration methods –

Trapezoidal rule, Simpson's rule and Romberg integration. (Sec 4.1-4.6, 4.9, 5.6-5.10 of Text book.1).

Unit-IV

Numerical techniques for solving ordinary differential equations: Euler method, backward Euler method, Midpoint method. Single step methods: Taylor series method, Runge-Kutta methods, Multistep methods: Predictor-corrector method, Adams Bashforth method, Adams –Moulton method, Convergence and stability analysis of single – step methods. (Sec 6.1-6.7 of Text book 1)

Unit-V

Numerical methods for solving elliptic partial differential equations: Difference methods, Dirichlet problem, Laplace and Poisson equations. (Sec 1.1, 1.2, 4.1, 4.2 of Text book 2).

Text books:

1. Numerical method for Scientific and Engineering Computation, M.K. Jain, S.R.K. Iyengar and R.K. Jain, 6rd edition, 2012, New Age International Pvt. Ltd.
2. Computational methods for partial differential equations by M.K. Jain, S.R.K. Iyengar and R.K. Jain, New Age International Pvt. Ltd. (1993).

M.Sc Degree Examination

Second Semester

Applied Mathematics

21AM205: Numerical Analysis

(Effective from the admitted batch of 2021-2022)

Time: 3 hours Maximum: 80 marks

Answer one question from each unit. All questions carry equal marks

UNIT-I

1. (a) Using Chebyshev method, find the root of the equation
   $f(x) = \cos x - xe^x = 0$ correct to six decimal places.
   (b) Derive the rate of convergence of Newton-Raphson Method.

OR

2. (a) Perform three iterations of the Bisection method to find the smallest
   positive root of the equation $f(x) = x^3 - 5x + 1 = 0$.
   (b) Derive the rate of convergence of Regula-Falsi Method.

UNIT-II

3. (a) Find the inverse of the coefficient matrix of the system

$$x_1 + x_2 + x_3 = 1$$

$$4x_1 + 3x_2 - x_3 = 6$$
$$3x_1 + 5x_2 + 3x_3 = 4$$

by Gauss-Jordan Method.

(b) Solve the system of equations

$$x_1 + 2x_2 + 3x_3 = 5$$
$$2x_1 + 8x_2 + 22x_3 = 6$$
$$3x_1 + 22x_2 + 82x_3 = -10$$

by Cholesky Method.

OR

4. (a) Solve the system of equations

$$4x_1 + x_2 + x_3 = 2$$
$$x_1 + 5x_2 + 2x_3 = -6$$
$$x_1 + 2x_2 + 3x_3 = -4$$

using Jacobi iteration method and its residual approach. Taking the initial approximation $x^{(0)} = (0.5, -0.5, -0.5)^T$, perform three iterations in each case.

(b) Find the Eigen values and Eigen vectors of the matrix $\begin{bmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{bmatrix}$

using Jacobi Method.

UNIT-III

5. (a) Construct the Hermite interpolating polynomial that fits the data

x	0.0	0.5	1.0
$f(x)$	0	0.4794	0.8415
$f'(x)$	1	0.8776	0.5403

Estimate the value of $f(0.75)$.

(b) Find the approximate value of the integral $I = \int_0^1 \frac{dx}{1+x}$ using composite Simpson's 1/3rd rule with 3,5,9 nodes and Romberg integration.

OR

6. (a) Determine the least square approximation of the type $ax^2 + bx + c$ to The function 2^x at the points $x_i = 0,1,2,3,4$.

(b) Fit the following four points by cubic splines:

X	1	2	3	4
y	1	5	11	8

use the end conditions $y''(1) = 0 = y''(4)$. Hence compute $y(1.5)$ and $y''(2.5)$.

UNIT-IV

7. (a) Solve the IVP $u' = t^2 - u^2, u(0) = 1, t \in [0, 0.6]$ using 3rd order Adam's Bashforth method with $h = 0.1$. Take the starting value using 3rd order Taylor series Method.
   (b) Solve the IVP $u' = \sqrt{t+u}$, $u(0.4) = 0.41$, $t \in [0.4, 0.8]$ with $h = 0.2$ using classical Runge-Kutta 4th order Method.

OR

8. (a) Solve the IVP $u' = -u^2, u(1) = 1$ using Euler Method. Compute $u(1.2)$ with $h = 0.1$.
   (b) Solve the IVP $u' = -2tu^2, u(0) = 1$ with $h = 0.2$ on [0, 0.4] using the $P - C$ Method.

$$P: u_{j+1} = u_j + \frac{h}{2}(3u_j' - u_{j-1}')$$
$$C: u_{j+1} = u_j + \frac{h}{2}(3u_{j+1}' + u_j') \text{ as } P(EC)^2E.$$

UNIT-V

9. (a) Find the characteristics of the following equation and reduce it to the appropriate canonical form $u_{xx} - 4u_{xy} + 4u_{yy} = \cos(2x + y)$.
   (b) Find the solution of $u_{xx} + u_{yy} = 0$ in R subject to the condition $u(x, y) = x - y$ on the boundary ∂R, where R is the region inside the triangle with vertices (0,0), (7,0), (0,7) using five point formula. assuming uniform step length $h = 0.2$ along the axes.

OR

10.(a) Classify the following PDE and reduce it to its appropriate canonical form

$$u_{xx} - xu_{yy} = 0 .$$

(b) Find the solution of $u_{xx} + u_{yy} = x + y$ in R subject to the condition $u(x, y) = \frac{x^2+y^2}{2}$ on the boundary ∂R, where R is a triangle $0 \le x \le 1, 0 \le y \le 1, 0 \le x + y \le 1$ using five point formula. Assuming uniform step length $h = \frac{1}{4}$ along the axes.

M.Sc. APPLIED MATHEMATICS (THIRD SEMESTER)

21AM-301: MEASURE THEORY

(With effect from 2021-2022 Admitted Batch)

Course Code:21AM301

(A total of Ten questions to be set by selecting two questions from each unit with internal choice. Each question carries 16 marks.)

Course outcomes (COs): At the end of the course, the students will be able to :

CO1: Apply the knowledge of concepts of functions of several variables and measure theory in order to study theoretical development of different mathematical concepts and their applications.
CO2 :Understand the nature of abstract mathematics and explore the concepts in further details.
CO3:Recognize the need of concept of measure from a practical view point.
CO4: Understand measure theory and integration from theoretical point of view and apply its tools in different fields of applications.
CO5: Extend their knowledge of Lebesgue theory of integration by selecting and applying its tools for further research in this and other related areas.

Course Specific outcomes (CSOs):
CSO1: Learn about measuring an object through the concept of outer measure, measurable set, non-measurable set.
CSO2: Learn Lebesgue integral of a function**.**
CSO3: Learn about Banach space and properties.

Learning outcomes(LOs):

LO1: By the end of the course the student is familiar with the basic concepts and results of Lebesgue measure theory (outer measure, measurable sets and connections with topology, Borel sigma algebra) as well as of Lebesgue theory of integrals (measurable functions, convergence theorems).
LO2 : The student masters' basic concepts from measure theory, including sets of measure zero, measurable functions, the Lebesgue integral and Lebesgue spaces.
LO3: The student has an overview of the central results of the theory of Lebesgue integration, including convergence theorems and Fubini's theorem.
LO4: They will learn about the classical Banach space and Its properties.
LO5: Existence of functionals on Banach spaces.

Mapping of course outcomes with the program outcomes:

	PO1	PO2	PO3	PO4	PO5
CO1	✓	✓	✓	✓	
CO2	✓		✓	✓	
CO3	✓	✓	✓		
CO4	✓	✓		✓	
CO5	✓	✓	✓	✓	

Unit-I

Lebesgue Measure: Introduction to Outer measure, Measurable sets and Lebesgue measure, A nonmeasurable set, Measurable functions, Littlewood's three principles. (Chapter 3 of Text book)

Unit-II

The Lebesgue Integral: The Riemann integral, The Lebesgue integral of a bounded function over a set of finite measure. The integral of a nonnegative function. The general Lebesgue integral, Convergence in measure. (Chapter 4 of Text book).

Unit-III

Differentiation: Differentiation of Monotone functions, Functions of bounded variation. (Section 1, 2 of Chapter 5 of Text book)

Unit-IV

Integration: Indefinite integral of a function, Absolute continuity, convex functions. (Section 3, 4 of Chapter 5 of Text book)

Unit-V

The classical Banach Spaces: The L_p spaces, The Holder and Minkowski inequalities, Convergence and completeness, Bounded linear functionals on the L_p spaces. (Chapter 6 of Text book)

<u>Text Book</u>: Real Analysis, H.L. Royden – Macmillan publishing Comp.

M.Sc Degree Examination
Third Semester
Applied Mathematics
21AM301: Measure Theory
(Effective from the admitted batch of 2021-2022)

Time: 3 hours Maximum: 80 marks

Answer one question from each unit. All questions carry equal marks.

UNIT-I

1. (a) Prove that the outer measure of an interval is it's length.

   (b)Let f be an extended real valued function whose domain D is measurable . Then prove that the following are equivalent

   (i) For each real number α the set$\{ x: f(x) > \alpha\}$ is measurable.

   (ii) For each real number α the set$\{ x: f(x) \geq \alpha\}$ is measurable.

OR

2. (a) If f is a measurable function and $f = g$ a.e, then show that g is measurable.

   (b) Let (E_n) be an infinite decreasing sequence of measurable sets with $E_{n+1} \subset E_n$ for each n. Let $m(E_1) < \infty$, Then prove that $m(\bigcap_{i=1}^{\infty} E_i) = \lim_{n\to\infty} m(E_n)$.

UNIT-II

3. (a)State and prove bounded convergence theorem for sequence of measurable functions.

   (b) If ϕ *and* Ψ are simple functions which vanish outside a set of finite measure , then show that $\int(\alpha\phi + \beta\Psi) = \alpha\int\phi + \beta\int\Psi$, where α, β are constants.

OR

4. (a) Let f be a bounded measurable function defined on a set E of finite measure. Let A and B are disjoint measurable sets of finite measure, then prove that

$$\int_{A\cup B} f = \int_A f + \int_B f$$

   (b) State and prove Lebesgue convergence theorem.

UNIT-III

5. (a)State and prove Vitali covering lemma.

   (b) Let f and g be non-negative and continuous at c. Then show that $D^+(fg)(c) \leq f(c)D^+g(c) + g(c)D^+f(c)$.

OR

6. (a)Prove that a function f is of bounded variation on [a, b] iff $f$is the difference of two monotone real valued functions on [a, b].

   (b) Let $f: [a, b] \to R$ and $g: [a, b] \to R$ and $x \in [a, b]$. Then show that

$V_a^x(f+g) \leq V_a^x(f) + V_a^x(g)$ and $V_a^x(cf) = |c|V_a^x(f)$ for some constant $c \in R$.

UNIT-IV

7. (a) Let f be an increasing real valued function on [a, b]. Then show that f is differentiable almost everywhere and also show that f' is measurable and $\int_a^b f'(x)dx \leq f(b) - f(a)$.

   (b) Let f be integrable on $[a,b]$ and $\int_a^x f(t)dt = 0\ for\ all\ x \in [a,b]$, then show that $f(t) = 0\ a.e\ in\ [a,b]$.

OR

8. (a) State and prove Jensen's Inequality.

   (b) If f is absolutely continuous on $[a,b]$and $f'(x) = 0\ a.e,$ then show that f is constant.

UNIT-V

9. (a) State and prove Riesz- representation theorem for bounded linear functionals.

   (b) If $f \in L^p$ and $g \in L^q$, then prove that $f.g \in L^1$and $\int|fg| \leq ||f||_p.||g||_q$, where p, q are non negative extended real numbers such that $\frac{1}{p} + \frac{1}{q} = 1$.

OR

10. (a) Prove that L^p spaces are complete , $1 \leq p < \infty$.

    (b)Let g be an integrable function on [0, 1] and $|\int fg| \leq M||f||_p$ for some constant M and all bounded measurable functions f. Then prove that $g \in L^q$.

M.Sc. APPLIED MATHEMATICS(THIRD SEMESTER)
21AM 302: PROGRAMMING IN PYTHON
(With effect from 2021-2022 Admitted Batch)
Course Code:21AM302

(A total of ten questions are to be set taking two questions from each unit with internal choice in each unit. Each question carries 16 marks.)

Course Outcome(COs):

CO1: This course introduces computer programming using the Python programming language which will help you to master the Programming with Python.

CO2: Introducing the Object Oriented programming concepts, creation of Data Structures.

CO3: Acquiring the basic knowledge of writing scripts using Python libraries and OOPs concepts.

CO4: Lastly you will get into design, code, test, and debug Python programming Language Scripts.

CO5: Increases the subject knowledge that helps in pursuing higher studies as well as getting employment.

Course Specific Outcome(CSOs):

CSO1: Study on the basic concepts ofobject oriented programming, data structures and various Python libraries.

CSO2: Introduction to Scripting Language.

CSO3: Exposure to various problems solving approaches of Applied Mathematics, computer science and information Technology.

Learning Outcome(LOs):

LO1: After studying the course, the students are expected to learn some basic concepts of object oriented programming, data structures and various Python libraries.

LO2: Learns to apply data structures concepts for programs and scripts writing using Python programming.

LO3: Gains knowledge and orientation for applying the knowledge for Python programming.

LO4: Helps in getting employment in various software companies as well as in higher educational institutions.

Mapping of course outcomes with the program outcomes:

	PO1	PO2	PO3	PO4	PO5
CO1	✓	✓		✓	
CO2			✓		
CO3	✓		✓		
CO4	✓	✓		✓	✓
CO5		✓		✓	✓

Unit 1. Introduction and fundamentals of Python

Introduction: What Is a Program?, Programming Languages, Installing Anaconda Python Distribution, Installing PyCharm IDE to Set Up a Python Development Environment Creating and Running Your First Python Project, Installing and Using Jupyter Notebook, Open Source Software

Parts of Python Programming Language, Identifiers, keywords, Statements and Expressions, Variables, Operators, Precedence and Associativity, Data Types, Indentation, Comments, Reading Input, Print Output, Type Conversions, The type() Function and Is Operator, Dynamic and Strongly Typed Language

Unit 2.Control Flow and functions

Control Flow: if, if…else , if…elif…else, Nested if Statement, The while Loop, The for Loop, The continue and break Statements, Catching Exceptions Using try and except Statement

Functions: Built-In Functions, Commonly Used Modules, Function Definition and Calling the Function, The return Statement and void Function, Scope and Lifetime of Variables, Default Parameters, Keyword Arguments, *args and **kwargs, Command Line Arguments

Unit 3.Strings and lists

Strings: Creating and Storing Strings, Basic String Operations, Accessing Characters in String by Index Number, String Slicing and Joining, String Methods, Formatting Strings

Lists: Creating Lists, Basic List Operations, Indexing and Slicing in Lists, Built-In Functions Used on Lists, List Methods,The del Statement, Use of numpy and pandas: basics

Unit 4.Dictionaries, Tuples and Sets

Dictionaries: Creating Dictionary, Accessing and Modifying key:value Pairs in Dictionaries, Built-In Functions Used on Dictionaries, Dictionary Methods,The del Statement

Tuples and Sets: Creating Tuples, Basic Tuple Operations, Indexing and Slicing in Tuples, Built-In Functions Used on Tuples, Relation between Tuples and Lists, Relation between Tuples and Dictionaries, Tuple Methods, Using zip() Function, Sets, Set Methods, Traversing of Sets, Frozenset

Unit 5.Files and OOP fundamentals

Files: Types of Files, Creating and Reading Text Data, File Methods to Read and Write Data, Reading and Writing Binary Files, OS paths.

OOP fundamentals: classes and objects, constructors, encapsulation, inheritance and polymorphism

TextBook

1) Introduction to Python Programming*Gowrishankar S.* and *Veena A.* and published by CRC Press/Taylor and Francis, Boca Raton, USA.

Reference Books:

1) Fundamentals of Python First programs, K Lambert et al, Cengage

2) Python Programming: A modern approach, VamsiKurama, Pearson

M.Sc Degree Examination

Third Semester

Applied Mathematics

21AM302: Python Programming

(Effective from the admitted batch of 2021-2022)

Time: 3 hours Maximum: 80 marks

Answer one question from each unit. All questions carry equal marks

UNIT- I

11) a) Give a brief explanation of the history of Python.
    b) Explain type conversion in python with examples.

OR

12) a)Discuss about the operators used in Python language.
    b) Write a program to convert temperature from Centigrade (read it as float value) to Fahrenheit.

UNIT-II

13) a)Differentiate the syntax of if...else and if...elif...else with an example.
    b) Write a program to check whether a number is prime or not using if blocks.

OR

14) a)Explain about *args and **kwargs. Write a python program to demonstrate their use.
    b) Write a program to find the largest of three numbers using functions.

UNIT-III

15) a) Explain the basic string operations in Python with examples.
    b)Write Python program to count the total number of vowels, consonants and blanks in a string.

OR

16) a) With the help of an example explain the concept of nested lists. Explain the ways of indexing and slicing the list with examples.
    b) Write Python program to add two matrices.

UNIT-IV

17) a)Define a dictionary. What are the advantages of using dictionary over lists.

b)Write a Python program to input information for n number of students as given below:

I. Name

II. Registration Number

III. Total Marks

The user has to specify a value for n number of students. The program should output the registration number and marks of a specified student given his name.

.

OR

18) a) Explain the relation between Tuples and Dictionaries.

b) Write Python program to swap two numbers without using intermediate/temporary variables.

UNIT-V

19) a) Explain the different file mode operations with examples.

b)Explain the ways to read and write binary files.

OR

20) a)Examine the different types of inheritances with an example

b) Given a point(x, y), write Python program to find whether itlies in the first, second, third or fourth quadrant of $x - y$ plane.

M.Sc. APPLIED MATHEMATICS (THIRD SEMESTER)

21AM-303: TECHNIQUES OF APPLIED MATHEMATICS

(With effect from 2021-2022 Admitted Batch)

Course Code:21AM303

(A total of Ten questions to be set by selecting two questions from each unit with internal choice. Each question carries 16 marks.)

Course outcomes(COs):

CO1: Familiar with the theory and applications of difference calculus, first order homogeneous difference equations .

CO2: Familiar with theory of non homogeneous difference equations and applications.

CO3: Familiar with theory and applications of system of difference equations.

CO4: Familiar with discrete transformations and its applications.

CO5: Familiar with optimizations by the method of calculus of variations.

Course Specific Outcomes(CSOs):

CSO1: Theory and applications of homogeneous and non homogeneous difference equations

CSO2: System of difference equations and its applications to certain models.

CSO3: Utilizing the applications of Z-transforms in solving difference equations.

Learning Outcomes(LOs):

LO1: Able to formulate and solving the homogeneous difference equations

LO2: Able to apply the theory of non homogeneous difference equation.

LO3: Able to find the relation between higher order difference equations and system of equations

LO4: Able to apply the discrete transformations in solving difference equations.

LO5: Able to apply the tools in calculus of variations in optimizing certain physical problems.

Mapping of course outcomes with the program outcomes:

	PO1	PO2	PO3	PO4	PO5
CO1	✓	✓	✓	✓	
CO2	✓	✓		✓	
CO3	✓	✓	✓		
CO4	✓		✓	✓	
CO5	✓	✓	✓	✓	

Unit-1

Difference equations: Introduction, Linear first order difference equations, Important special cases, Basics of difference calculus, General theory of difference equations, first order difference equations explained through examples, steady-state and stability, linear homogeneous equations with constant coefficients. (Chapter 1-sections 1.1, 1.2 & Chapter 2 – sections 2.1 to 2.3 of Text book 1)

Unit-II

Non-homogeneous equations, method of undetermined coefficients, Limiting behavior of solutions, Non-linear equations transformable to linear equations system of linear difference equations, Applications. (Chapter 2 – sections 2.4 to 2.7 of Text book 1)

Unit-III

System of linear difference equations : Autonomous systems, Discrete analogue of the Putzer algorithm, Algorithm for A^n, The basic theory, The Jordan form, Linear periodic systems. (Chapter 3-sections 3.1 to 3.4 of Text book 1)

Unit-IV

Z-transform methods, definition with examples, properties of Z-tranforms, inverse Z-transforms, solution to difference equations by Z-transform method. (Chapter 6-Section 6.1, 6.2of Text book 1),

Unit-V

Calculus of variations : Euler's equations, functions of the form $\int_{x_0}^{x_1} F(x, y_1, y_2 \dots y_n, y_1', y_2' \dots y_n')dx$. Functional dependence on higher order derivatives, variational problems in parametric form and applications (Chapter VI of Text book 2).

Text books:

1. An introduction to difference equations by Saber Elaydi, Springer Publisher, Third Edition.
2. L. Elsgolts: Differential equations and calculus of variations, Mir Publishers, Moscow.

M.Sc Degree Examination

Third Semester

Applied Mathematics

21AM303:Techniques of Applied Mathematics

(Effective from the admitted batch of 2021-2022)

Time: 3 hours | Maximum: 80 marks

Answer one question from each unit. All questions carry equal marks

UNIT-I

1.(a) Find the solution of the difference equation,

$$x(n+3) + 3x(n+2) - 4x(n+1) - 12x(n) = 0.$$

(b) Show that the operators Δ and Δ^{-1} are linear.

OR

2. (a)(i) If $p(E) = a_0E^k + a_1E^{k-1} + \cdots + a_kI$; where E is the shift operator and $g(n)$ is a discrete function, prove that $(E)\big(b^n g(n)\big) = b^n p(bE)g(n)$.

(ii) For fixed $k \in \mathbb{Z}^+, x \in \mathbb{R}$, prove that $\Delta x^{(k)} = kx^{(k-1)}$, where Δ is forward difference operator.

(b)State and prove Abel's lemma for difference equations.

UNIT-II

3.(a)Solve the initial value problem

$y(n+2) - 5y(n+1) + 6y(n) = 2^n, y(1) = y(2) = 0$ by the method of variation of parameters.

(b) Solve the difference equation $x(n+1) = \frac{2x(n)+3}{3x(n)+2}$.

OR

4. (a) Explain Pielou logistic equation and derive it's solution.

(b) Find the conditions under which the solutions of the equation

$y(n+2) - \alpha(1+\beta)y(n+1) + \alpha\beta y(n) = 1, \alpha, \beta > 0.$

(i) Converge to the equilibrium point y^*, and

(ii) Oscillate about y^*.

UNIT-III

5. (a) Find A^n if A is given by $\begin{bmatrix} 0 & 1 & 1 \\ -2 & 3 & 1 \\ -3 & 1 & 4 \end{bmatrix}$.

(b) Find the general solution of the system of difference equations

$x_1(n+1) = -x_1(n) + x_2(n)$; $x_2(n+1) = 2x_2(n)$ with $x_1(0) = 1$; $x_2(0) = 2$ by discrete Putzer algorithm.

OR

6. (a) Prove that for every fundamental matrix $\phi(n)$ of the system

$x(n+1) = A(n)x(n)$, there exists a non-singular periodic matrix $P(n)$ of period N such that $\phi(n) = P(n)B^n$.

(b) Prove that the particular solution of $y(n+1) = A(n)y(n) + g(n)$, $y_p(n_0) = 0$ is

$y_p(n) = \sum_{r=n_0}^{n-1} \phi(n, r+1)g(r)$.

UNIT-IV

7. (a) Solve the difference equation $x(n+2) + 3x(n+1) + 2x(n) = 0$

with $x(0) = 1, x(1) = -4$ by Z-transform method.

(b) Find the Z transform and its radius of convergence of the sequence

$g(n) = a^n \cos(\omega n)$.

OR

8. (a) Find the inverse Z-transform of $\tilde{x}(z) = \frac{z(z-1)}{(z-2)^2(z+3)}$.

(b) Let R be the radius of convergence of $\tilde{x}(z)$. Show that

$z\,[n^k x(n)] = (-z\frac{d}{dz})^k\, \tilde{x}(z)$ for $|z|>R$.

UNIT-V

9. (a)Derive Euler's equation for minimizing the functional

$v[y(x)] = \int_{x_0}^{x} f(x, y, y')\, dx$ where $y(x_0) = y_0$, $y(x_1) = y_{1,}$.

b)On what curves can the functional $v[y(x)] = \int_0^1 [y'^2 + 12xy]dx, y(0) = 0, y(1) = 1$ be extremized.

OR

10. (a) Find the curve connecting given points A and B which is traversed by a particle sliding from A to B in the shortest time .

(b) Find the extremals of the functional

$$v[y(x)] = \int_{x_0}^{x_1} \left[y''^2 - 2y'^2 + y^2 - 2y\sin x\right] dx.$$

M.Sc. APPLIED MATHEMATICS (THIRD SEMESTER)

21AM 304(A): BOUNDARY VALUE PROBLEMS-I

(With effect from 2021-2022 Admitted Batch)

Course Code:21AM304(A)

(A total of Ten questions to be set by selecting two questions from each unit with internal choice. Each question carries 16 marks.)

Course Outcomes(COS) :

CO1: The quantitative properties theory on Differential equations taught in detailed. Existence and Uniqueness of solution for Initial value problem and certain results on solutions dependence on initial conditions and parameters to be learn.

CO2: The system of first order homogeneous and non homogeneous equations and their solution space are discussed.

CO3: The adjoint vector and scalar equations and their relations are to be established. A procedure to determine Fundamental matrix to be developed for various types of eigenvalues.

CO4: The homogeneous and non homogeneous two point boundary value problems and their solutions in terms of Green's matrix/function to be learned.

CO5: As an application Controllability and observability of the system to be taught in detailed.

Course Specific Outcomes(CSOs):

CSO1: The relation between higher order differential equations and system of equations and results on them are discussed.

CSO2: The Boundary value problems and their solutions for system of equations and higher order scalar differential equations , scalar and vector adjoint boundary value problems are discussed.

CSO3: Linear control systems, Controllability and observability of the systems and its applications to real world problems are discussed.

Learning Outcomes(LOs):

LO1:Familiar with the quantitative properties of differential equations like what are the conditions are required for the existence and uniqueness of solutions of initial value problems and their properties.

LO2:Various results on adjoint vector , scalar equations and determining the fundamental matrix of the system of equations are to be familiar.

LO3: Understanding the Results on Homogeneous Boundary value problems associated with vector and higher order differential equations and their index of compatibility.

LO4:Able to express the solutions of non homogeneous boundary value problems associated with vector and scalar equations in terms of Green's matrix and Green's function and familiar with properties of above functions.

LO5: Familiar with the concepts of Controllability and observability and their applications.

Mapping of course outcomes with the program outcomes

	PO1	PO2	PO3	PO4	PO5
CO1	✓		✓	✓	
CO2	✓		✓	✓	
CO3	✓		✓	✓	
CO4	✓		✓	✓	
CO5	✓		✓	✓	

UNIT - I

Elementary Topology on Metric spaces: Mappings on metric spaces, Existence and uniqueness theorem via the principle of contraction. Continuation of solutions, Dependence of solutions on initial conditions and on parameters.

(Chapter 2 of text book I)

UNIT - II

General theory for linear first order system of equations, solution space, The non-homogeneous equation. The nth order linear homogeneous equation, The nth order non-homogeneous equation, The adjoint vector equation, The adjoint nth order equation, The relationship between scalar and vector adjoints. **(Chapter 3 of text book l)**

UNIT - III

Linear equation with constant coefficients, Real distinct eigenvalues, The general solution. Direct solutions, Real solutions associated with complex eigenvalues. The two point boundary value problem: The two point homogeneous boundary value problem, the adjoint boundary value problem.

UNIT - IV

The non-homogeneous boundary value problem and Green's matrix. The nth order boundary value problem, The nth order adjoint boundary value problem, the nth order non-homogeneous boundary value problems and Green's function. Self-adjoint boundary value problem

(Chapter 4, Section: 4.1, 4.2, 4.3, and Chapter 6 of text book l)

UNIT - V

Linear Control System: Controllability, Observability, Controllability and Polynomials, linear feed back, state observers, Relization of constant systems.

(Chapter 4 of text book 2)

Text books:

1. Theory of Ordinary differential equations, Randal H.Cole Appleton-Century-Crafts, New York (1968)

2. Introduction to Mathematical Control Theory, S.Barnett, R.G.Camaron, Clarendon Press, 1985.

Reference book: Theory of Ordinary differential equations by E.A. Coddington and Normal Levinson, Tata Mcgraw Hill Inc., New York (1980)

M.Sc Degree Examination

Third Semester

Applied Mathematics

21AM304(A):Boundary Value problems-I

(Effective from the admitted batch of 2021-2022)

Time: Three hours Maximum: 80 marks

Answer one question from each unit

All questions carry equal marks

Unit-I

1. (a) Prove that the solutions of IVP depends continuously on initial conditions.

   (b) Find the solution of $y' = \begin{pmatrix} 0 & 1 \\ -\lambda^2 & 0 \end{pmatrix} \mathrm{y},\ \mathrm{y}(\tau) = \bar{b}$.

(or)

2. (a) State and prove existence theorem for the solution of system of equations

$$y' = \mathrm{f(x, y)}, \qquad \mathrm{y}(t_0) = y_0\,.$$

   (b) Obtain a sequence of vectors which converges to solution for the IVP

$$y' = \begin{pmatrix} 0 & 1 \\ -2 & 3 \end{pmatrix} \mathrm{y},\ \mathrm{y}(0) = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

Unit-II

3. (a) Establish the relationship between the solution of scalar and vector adjoints.

   (b) Find the particular solution of the equation system $y' = \begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix} y + \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

(or)

4.(a) If u is any vector whose components are linearly independent functions in $c^n[a, b]$, then the differential equation for which u is a fundamental vector is $Ly = 0$, where $Ly = y^n - \widetilde{d_n}\ k'(u)k^{-1}(u)k(y)$.

   (b) Show that $p_0 u'' + p_1 u' + p_2 u = 0$ is self-adjoint if $p_1 = p_0{}'$.

Unit-III

5. (a) Find a fundamental matrix for $y' = \mathrm{A}\mathrm{y}$ where $A = \begin{pmatrix} 1 & 3 & 8 \\ -2 & 2 & 1 \\ -3 & 0 & 5 \end{pmatrix}$.

   (b) Find the index of compatibility and solution space of the boundary value problem

$$y' = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \mathrm{y}, \qquad \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \mathrm{y}(0) + \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix} \mathrm{y}(1) = 0.$$

(or)

6.(a) Determine the fundamental matrix for

$$y' = \begin{pmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{pmatrix} y + \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

(b) With usual notation find the adjoint boundary value problem for $y' = \mathrm{Ay}$,

$$w(a)y(a) + w(b)y(b) = 0.$$

Unit-IV

7. (a) Find a formula involving Green's matrix for the solution of $\mathrm{y}' = \mathrm{Ay} + \mathrm{f}$,

$$\mathrm{w}^{[a]}\mathrm{y}(a) + \mathrm{w}^{[\mathrm{b}]}\mathrm{y}(\mathrm{b}) = 0.$$

(b) State and prove the properties of the above Green's matrix.

(or)

8. (a) Find the Green's function to the boundary value problem

$$u'' = 0,$$
$$u(0) - u'(0)=0,$$
$$u(1) + u'(1)=0.$$

(b) Find the values of the parameter λ for which the boundary value problem $u'' + \lambda^2 \mathrm{u} = 0, \mathrm{u}(0) = 0, \mathrm{u}(\pi) = 0$ is compatible.

Unit-V

9. (a) Prove that the constant system $\dot{x} = Ax + Bu$ is completely controllable if and only if the $n \times nm$ controllability matrix $U = [B, AB,\ A^2B, \dots, A^{n-1}B]$ has rank n.

(b) Show that the system $\dot{x} = \begin{bmatrix} -2 & 2 \\ 1 & -1 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$ is completely controllable.

(or)

10 (a) Prove that the system $\dot{x} = A(t)x(t) + B(t)u(t),\ y = C(t)x(t)$ is completely observable if the symmetric observability matrix

$$V(t_0, t_1) = \int_0^t \Phi^T(\tau, t_0)\mathrm{C}^{\mathrm{T}}(\tau)\mathrm{C}(\tau)\Phi(\tau, t_0)\, d\tau$$

is non singular.

(b) Show that system $\dot{x}_1 = a_1x_1 + b_1u,\ \dot{x}_2 = a_2x_2 + b_2u,\ y = x_1$ is completely observable.

M.Sc. APPLIED MATHEMATICS(THIRD SEMESTER)
21AM305(B)OPTIMIZATION TECHNIQUES-I
(With effect from 2021-2022 Admitted Batch)
Course Code:21AM305(B)

(A total of ten questions are to be set taking two questions from each unit with internal choice in each unit. Each question carries 16 marks.)

Course Outcome(COs):

CO1: This course introduces some concepts of Operations Research some important optimization techniques which will help the students to master the skills in optimization.

CO2: Provides an in-depth knowledge of problem solving through various optimization techniques.

CO3: Tests and develops the students' knowledge of basic understanding of the problems through practical illustrations and examples and learn the basic concepts of Linear Programming, solution of LP problems.

CO4: Improves the logical thinking ability of the students and helps gain access to various employment opportunities.

CO5: Infuses practical knowledge that helps in pursuing higher studies as well as getting employment.

Course Specific Outcome(CSOs):

CSO1:Study on some optimization techniques like Linear Programming their solutions by simple methods.

CSO2:Gain knowledge of finding maximum or minimum for many real problems arising in Transportation, Assignment problems.

CSO3:Exposure to various problem solving approaches of dynamic programing and non-linear programming techniques.

Learning Outcome:

LO1: After studying this course, the students are expected to learn some techniques problem solving like simplex method, revised simplex method, transportation problem solving, assignment problem, etc.

LO2: Gains understanding of the practical problems and motivation for improvement.

LO3: Gains knowledge in various optimization techniques, dynamic programming, etc.

LO4: Gain ability to problem formulations, analysis and solution techniques.

LO5: Gain knowledge in computational skills and numerical problem solving.

Mapping of course outcomes with the program outcomes:

	PO1	PO2	PO3	PO4	PO5
CO1	✓	✓			
CO2			✓		
CO3			✓		
CO4	✓			✓	✓
CO5		✓		✓	✓

Unit-I

Linear Programming anits Applications: Formulation of L.P. problems, slack and surplus variables, convex sets, simplex method, artificial variables techniques, big M-method, degeneracy, revised simplex method. (Chapter I (expect 1,3),Chapter II, Chapter III, Chapter IV of unit 2 and Appendix – A of Unit-6 of text book 1)

Unit-II

Duality in linear programming, the dual simplex method, Integer linear programming, Gomory's cutting plane method, branch and bound method (Chapter V, Chapter VI and Chapter VIII of unit –2 of text book 1)

Unit-III

Assignment models, Hungarian method, the traveling salesman problem, transportation models, methods for initial basic feasible solutions, MODI method, degeneracy in transportation problems. (Chapter IX, Chapter X, of unit 2 of text book1)

Unit-IV

Dynamic programming, concepts of dynamic programming, Bellman's principle of optimality, simple models(7.1 to 7.9 of Chapter VII of unit 5 of text book 1)

Unit-V

Non-linear programming-One dimensional minimization methods: Fibonacci method, Golden section method, unconstrained optimization techniques: Hooke and Jeeves' method -descent methods-gradient of a function, steepest descent(Cauchy) method, conjugate gradient (Fletcher-Reeves) method, Newton method, Marquardt method, Quasi-Newton method
(Chapter 5 - 5.7, 5.8 Chapter 6- 6.6, 6.10 to 6.15 of text book 2)

Text book: 1.S.D. Sharma, Operations Research, KedarnathRamnath& Company.

2. Optimization Theory and Applications by S. S. Rao, Wiley Eastern Limited,second edition.

Reference book: Linear Programming by G. Hadley, Oxford, IBH publishing Co.

M.Sc Degree Examination

Third Semester

Applied Mathematics

21AM305(B): Optimization Techniques-I

(Effective from the admitted batch of 2021-2022)

Time: 3 hours Maximum: 80 marks

Answer one question from each unit. All questions carry equal marks

UNIT- I

21) a) Solve the LPP by simplex method:

$$Maximize\ z = 2x_1 + 6x_2 subjectto x_1 + 10x_2 \leq 7;\ 12x_1 + 3x_2 \leq 9;$$
$$3x_1 + 7x_2 \leq 11;\ x_1, x_2 \geq 0.$$

b) Solve the following LPP by Big-M method $Minimizez = 7x_1 + 6x_2$

$$Subjectto - x_1 + 9x_2 = 6;\ x_1 + 3x_2 \leq 9;\ 3x_1 - 7x_2 \geq 1, x_1, x_2 \geq 0.$$

OR

22) a) Use revised simplex method to solve the following LPP

$$Maximize\ z = 7x_1 + 2x_2 - 4x_3 subject\ to\ 6x_1 - 4x_2 + 5x_3 \leq 6;$$
$$x_1 + 2x_2 + 13x_3 = 10\ ;\ x_1, x_2, x_3 \geq 0.$$

b) Explain the special cases in LP problems. During simplex procedure, how do you detect them.

UNIT-II

23) a) Explain the primal dual relationships. Write down the dual of the following LPP:

Maximize $z = 4x_1 + 3x_2 - x_3 + 3x_4$

Subject to $44x_1 + x_2 + x_3 = 12$

$-x_1 + 5x_2 + 8x_3 - 9x_4 \leq 4$

$x_1, x_2, x_4 \geq 0$; and x_3 is unrestricted in sign.

b) Use dual simplex method to obtain the optimal basic feasible solution to the LPP:

Maximize $z = -4x_1 - 6x_2 - 18x_3$

Subject to $x_1 + 3x_3 \geq 3$

$x_2 + 2x_3 \geq 5$

$x_1, x_2, x_3 \geq 0$

OR

24) Define integer linear programming problem. Solve the following linear programming problem using Branch and Bound method

$$\text{Maximize } z = 4x_1 + 3x_2$$
$$\text{Subject to } 3x_1 + x_2 \leq 15$$
$$3x_1 + 4x_2 \leq 24$$

$x_1, x_2 \geq 0$; and are integers

UNIT-III

25) a) Explain the steps of MODI method and use it to find the optimum cost of the following transportation problem:

	T1		T2	T3	T4	Supply
A	26	14	10	12	22	
B	31	27	30	14	46	
C	15	18	16	25	12	
Demand	17	12	21	30		

b) Explain degeneracy in transportation problem and discuss the method to resolve it.

OR

26) a) Explain the mathematical formulation of assignment problem and describe the algorithm to solve the problem.

b) Solve the following assignment problem:

	I	II		III	IV	V
A	8	26	17	11	10	
B	13	28	4	26	12	
C	38	19	18	15	5	
D	19	26	24	10	8	

UNIT-IV

27) a) Use dynamic programming to solve

$\text{Maximize} z = y_1 y_2 y_3 \text{ S.T. } y_1 + y_2 + y_3 = 5,\ y_1,\ y_2,\ y_3 \geq 0.$

b) Explain the main characteristic features of dynamic programming.

OR

28) Explain the procedure for solving a LP problem using dynamic programming approach. Using Dynamic Programming, solve the following LPP

$$\text{Max } z = 3x_1 + 2x_2$$
$$\text{s.t. } x_1 + x_2 \le 300$$
$$2x_1 + 3x_2 \le 800$$
$$x_1, x_2 \ge 0.$$

UNIT-V

29) a) Explain Fibonacci method for solving unconstrained optimization problem.

b) Minimize $f(x_1, x_2) = x_1 - x_2 + 2x_1^2 + 2x_1x_2 + x_2^2$ starting from the point $X_1 = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$.

OR

30) Minimize $f = x_1^2 + 3x_2^2 + 6x_3^2$ by the Hooke-Jeeve's method by taking $\Delta x_1 = \Delta x_2 = \Delta x_3 = 0.5$ and the starting point as $(2, -1, 1)$. Perform two iterations.

M.Sc. APPLIED MATHEMATICS (THIRD SEMESTER)

21AM 306(C): RELATIVITY AND COSMOLOGY-I

(With effect from 2021-2022 Admitted Batch)

Course Code:21AM306(C)

(A total of ten questions are to be set taking two questions from each unit with internal choice in each unit. Each question carries 16 marks.)

Course Outcomes(COs): At the end of the course, the students will be able to

CO1: Describe the concept of Tensor and their properties.

CO2: Describe the concepts flat space and space of constant curvature.

CO3: Explain the meaning and significance of the postulate of Special Relativity.

CO4: Explain true nature of Lorentz transformation and their consequences.

CO5: Explain relativistic transformation equations for mass, work and kinetic energy.

Course Specific Outcomes(CSOs):

CSO1: Develops ability to solve mathematical problems involving vectors and tensors.

CSO2: Competently use vector and tensor algebra as a tool in the field of applied sciences and related fields.

CSO3: Provide advanced knowledge on topics in mathematical physics, empowering the students to pursue higher degrees at reputed academic/research institutions.

Learning Outcomes(LOs): Upon successful completion of this course, it is intended that a student will be able to

LO1: Know the fundamental mathematics of vector and tensor that are important for higher learning.

LO2: Understand the effect of co-ordinate transformations. Also, students shall learn the advanced concepts of tensor calculus, which will be useful in theory of relativity.

LO3: Demonstrate knowledge and broad understanding of Special Relativity.

LO4: Understand the meaning of relativity (as a coordinate symmetry) and the key role played by Einstein's new conception of space and time in its formulation.

LO5: Learn to use 4D tensors (such as 4- position, 4-derivative, 4-momentum, and electromagnetic field tensor and energy-momentum-stress tensor, etc.) to construct relativistic equations.

Mapping of course outcomes with the program outcomes					
	PO1	**PO2**	**PO3**	**PO4**	**PO5**
CO1	√	√	√	-	-
CO2	√	√	√	-	-
CO3	√	-	√	√	-
CO4	√	-	√	√	-

CO5	√	-	√	√	-

Unit-I

Tensor Analysis: N-dimensional space, covariant and contravariant vectors, contraction, second & higher order tensors, quotient law, fundamental tensor, associate tensor, angle between the vectors, principal directions, Christoffel symbols, covariant and intrinsic derivatives, geodesics (Chapters 1 to 4 of Text book.1).

Unit-II

Riemann Christoffel Tensor, covariant curvature tensor and its properties, Ricci Tensor, Curvature invariant, Einstein space, Bianchi's identity, Riemannian Curvature, Einstein space, flat space, space of constant curvature, Schur's Theorem (Chapter V of Text book.1).

Unit-III

Space-time continuum, the three plus one dimensions of space-time, the geometry corresponding to space-time, the signature of the line element and the three kinds of interval, Lorentz rotation of axes, transformation to proper coordinates (Chapter II, Articles 13-18 of Text book 2).

Unit-IV

The mass of a moving particle, the transformation equations for mass, work and kinetic energy, the relations between mass, energy and momentum, Four-dimensional expressions of the mechanics of a particle (Chapter III, Articles 23-28 of Text book 2).

Unit-V

The Maxwell-Lorentz Field Equations, The transformation equations for E, H and Q. The force on a moving charge, The energy and momentum of electromagnetic field, electromagnetic stresses, Four dimensional expressions for electron theory (Chapter IV, Articles 39-43 & 46 of Text book 2).

Text Books:

1. Barry Spain, Tensor Calculus-Radha Publishing House, Calcutta.
2. R.C. Tolman, Relativity, Thermodynamics and Cosmology, Clarendon Press, Oxford.

Books for further reference:

1. Introduction to Special Relativity by Robert Resnick, Johnwiley & Sons, New York.
2. Theory of Relativity by S.R. Roy and Raj bali Jaipur Publishing House, Jaipur.
3. J.K. Goyal and K.P. Gupta, Theory of Relativity, Krishna Prakasan Media(P) Ltd., Meerut.

M.Sc Degree Examination

Third Semester
Applied Mathematics
21AM306(C):Relativity & Cosmology-I
(Effective from the admitted batch of 2021-2022)

Time: 3 hours Maximum: 80 marks

Answer one question from each unit. All questions carry equal marks

Unit-I

1). (a) Sate and prove Quotient law of tensors.

(b) Show that the metric of a Euclidean space, referred to spherical polar coordinates $x^1 = r, x^2 = \theta$ and $x^3 = \psi$ is given by $ds^2 = dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta \ d\psi^2$.

(OR)

2). (a) If $A_{ij} = B_{i,\ j} - B_{j,\ i}$, prove that $A_{i\,j,\ k} + A_{j\,k,\ i} + A_{k\,i,\ j} = 0$.

(b) If A^{ijk} is skew symmetric tensor, show that $\frac{1}{\sqrt{g}}\frac{\partial}{\partial\, x^k}\left(\sqrt{g}\ \ A^{ijk}\right)$ is a tensor.

Unit-II

3). (a) Obtain an expression for Riemann Christoffel Tensor and also Ricci Tensor.

(b) Derive covariant curvature tensorR_{hijk} and discuss properties of covariant curvature tensor.

(OR)

4). (a) Sate and prove Bianchi's identity.

(b) Prove that $R_{1212} = -G\frac{\partial^2 G}{\partial u^2}$ for the V_2 whose line- element is $ds^2 = du^2 + G^2 dv^2$, where G is a function of u and v.

Unit-III

5). (a) Show that the Lorentz transformations about to rotation of axis in space time.

(b) Explain the following terms in detail.

(i) World line. (ii) Space-like vector. (iii) Light cone.

(OR)

6). (a) Write about Minkowski-Space.

(b) Explain the transformation to proper coordinates.

Unit-IV

7). (a) Derive expression for variation of mass of a body with velocity.

(b) Derive the transformation formula for force of a body.

(OR)

8). (a) Establish the relation $E = mc^2$ and discuss the equivalence of mass and energy.

(b) Prove that $E^2 = p^2c^2 + m_0^2c^4$ for all the particles in inertial frames.

Unit-V

9). (a) Show that the equation of continuity in electrodynamics in the form : $\frac{\partial \rho}{\partial t} + div\, J = 0$.

(b) Derive the transformation equations for electric and magnetic field intensities $\overline{E}$ and $\overline{H}$.

(OR)

10). Obtain Maxwell's equations in tensor form.

M.Sc. APPLIED MATHEMATICS (THIRD SEMESTER)

21AM 307(D): Numerical solution of Partial Differential Equations-I

(With effect from 2021-2022 Admitted Batch)

Course Code:21AM307(D)

(A total of Ten questions to be set by selecting two questions from each unit with internal choice. Each question carries 16 marks.)

Course outcomes (COs):

On completion of this course, the students will have the ability to

CO1: learn to make a connection between the mathematical equations or properties and the corresponding physical meanings.

CO2: learn the connection between the exact relationship between derivatives and finite difference operators (Delta operators)

CO3: learn the principles for designing numerical schemes, both explicit and implicit, based on finite difference methods for PDEs.

CO4: analyze the numerical methods (schemes) for consistency, stability and convergence of a numerical scheme.

CO5: know, for each type of PDEs (hyperbolic, parabolic and elliptic), what kind of numerical methods are best suited and the reasons behind these choices.

Course Specific outcomes (CSOs):

CSO1: Learn about Elliptic PDEs and finding its solution numerically.

CSO2: Learn about Parabolic PDEs and finding its solution numerically.

CSO3 : Learn about Hyperbolic PDEs and finding its solution numerically.

Learning Outcomes (LOs):

LO1: This course is designed to prepare students to solve mathematical models represented by initial or boundary value problems involving partial differential equations that cannot be solved directly using standard mathematical techniques.

LO2: Students are introduced to the discretization methodologies, with particular emphasis on the finite difference method, that allows the construction of accurate and stable numerical schemes.

LO3 :In depth discussion of theoretical aspects such as stability analysis and convergence will be used to enhance the students' understanding of the numerical methods.

LO4: They will be able to construct stable numerical methods for solving problems in science, engineering, option pricing.

LO5: They will be able to identify different methods for different type of PDEs.

Mapping of course outcomes with the program outcomes:

	PO1	PO2	PO3	PO4	PO5
CO1	✓	✓	✓	✓	✓
CO2	✓	✓	✓	✓	✓
CO3	✓	✓	✓	✓	✓
CO4	✓	✓	✓	✓	✓
CO5	✓	✓	✓	✓	✓

Unit-I

Numerical solutions of ODEs: General feature of Initial-Value ODEs. Basic Discretization methods: Taylor series method, Euler methods- Explicit and Implicit approaches. Consistency, stability, and convergence. (Chapter 7 of Textbook 1)

Unit-II

Numerical solutions of elliptic PDEs: General feature of elliptic PDEs. Finite difference approximation of Laplace equation. Consistency, and convergence. Iterative methods of solutions. Finite difference approximation of Poisson equation. (Chapter 9 of Textbook 1; Chapter 4 of Textbook 2)

Unit-III

Numerical solutions of parabolic PDEs: General feature of parabolic PDEs, Classification of 2^{nd} order PDEs in two independent variables via Characteristics. Parabolic equations in 1-D: Explicit and implicit finite difference schemes, Truncation error and consistency, Stability analysis (matrix method, maximum principle, Fourier analysis). (Chapter 11 of Textbook 1; Chapter 2 of TextBook 2)

Unit-IV

Parabolic Problems in 2-Dimension, Maximum principle and convergence, Lax equivalence theorem, general boundary conditions, multilevel difference schemes, explicit and implicit methods, ADI methods. (Chapter 11 of Textbook 1; Chapter 2 of TextBook 2)

Unit-V

Numerical solutions of hyperbolic PDEs: General feature of hyperbolic PDEs. Method of characteristics. FTCS, upwind, Lax, BTCS, Lax-Wendroff methods. Consistency, stability, and convergence. (Chapter 12 of Textbook 1; Chapter 3 of Textbook 2)

Text Books:

1. J. D. Hoffman, Numerical methods for Engineers and Scientists, McGraw Hill, 1993.
2. M.K. Jain, S.R.K. Iyengar, Computational Methods for Partial Differential Equations, New Age International, 2005.

Reference Books:

1. G. D. Smith, Numerical Solutions to Partial Differential Equations: Finite Difference Methods, Oxford University Press, 3rd Edn., 1986.
2. K. W. Morton and D. F. Mayers, Numerical Solution of Partial Differential Equations, Cambridge University Press, 2nd Edn., 2005
3. C. Strikwerda, Finite Difference Schemes and Partial Differential Equations, SIAM, 2004.
4. R. L. Burden and J. D. Fairs, Numerical Analysis, Brooks/Cole, 2001

LAB - PYTHONPROGRAMMING

Code:21AMPR301

Course Outcome:

CO1: This course introduces computer programming using the Python programming language which will help you to master the Programming with Python.

Learning Outcome:

LO1: Learns to apply different concepts of data structures and apply them for preparing modules/scripts using Python programming.

LIST OF PROGRAMS

1. Write a program to compute HCF/GCD of two given numbers.

2. Write a program to check whether a number is prime or not .
3. Write a program to find the largest of n numbers using functions.
4. Write a program to convert binary to decimal number.
5. Write a program to generate and print the Fibonacci sequence.
6. Write Python program to count the total number of vowels, consonants and blanks in a given string.
7. Write a program to print the given number in reverse order and test for palindrome.
8. Write a python program to find the factorial of a given number using recursive function.
9. Write Python program to sorting of numbers in ascending or descending order.
10. Program for generating reports for student name with marks using lists.
11. Python program to solve quadratic equation.
12. Program to find the transpose of a matrix.
13. Program to find the product of two matrices.
14. Write a Python program to create a dictionary.

M.Sc. APPLIED MATHEMATICS (FOURTH SEMESTER)

21AM-401: FUNCTIONAL ANALYSIS
(With effect from 2021-2022 Admitted Batch)
Course Code:21AM401

(A total of Ten questions to be set by selecting two questions from each unit with internal choice. Each question carries 16 marks.)

Course outcomes (COs): At the end of the course, the students will be able to

CO1: Understand the concepts of topological spaces and the basic definitions of open sets, neighbourhood, interior, exterior, closure and their axioms for defining topological space. Understand the concept of Bases and Subbases, create new topological spaces by using subspace.

CO2: Understand continuity, compactness, connectedness, homeomorphism and topological properties. Understand how points of space are separated by open sets, Housdroff spaces and their importance. Understand regular and normal spaces and some important theorems in these spaces

CO3: Explain the fundamental concepts of functional analysis and their role in modern mathematics. Utilize the concepts of functional analysis, for example continuous and bounded operators, normed spaces, Hilbert spaces and to study the behaviour of different mathematical expressions arising in science and engineering.

CO4: Understand and apply fundamental theorems from the theory of normed and Banach spaces including the Hahn-Banach theorem, the open mapping theorem, the closed graph theorem and uniform boundedness theorem.

CO5: Understand the nature of abstract mathematics and explore the concepts in further details. Explain the concept of projection on Hilbert and Banach spaces.

Learning Outcomes(LOs):

LO1: Students will be able to understand the topological-algebraical structures of the spaces.

LO2: the main properties of bounded operators between Banach and Hilbert spaces.

LO3: the basic results associated to different types of convergences in normed spaces.

LO4: the spectral theorem and some of its applications.

LO5: With this knowledge they will be able to correlate Functional Analysis to problems arising in Partial Differential Equations, Measure Theory and other branches of Mathematics.

Course Specific outcomes (CSOs):

CSO1: Learn about topological space, its base and subbase construction of new topology from old ones.

CSO2: Learn about Banach space and its properties.

CSO3: Learn about Hilbert space, operators on this space, spectral theorem.

Mapping of course outcomes with the program outcomes:

	PO1	PO2	PO3	PO4	PO5
CO1	✓	✓	✓	✓	✓
CO2	✓	✓	✓	✓	✓
CO3	✓	✓	✓	✓	✓
CO4	✓	✓	✓	✓	✓
CO5	✓	✓	✓	✓	✓

Unit-I

Topological spaces: Elementary concepts, open bases and open subbases, weak topologies, function algebras Co (X, R) and Co (X, C), compact spaces, product spaces, Tychonoff's theorem, separation concepts. (Section 16-23, 26, 27 of Text book)

Unit-II

Banach spaces: Definition and some examples, continuous linear transformations, the Hahn-Banach theorem, the natural imbedding of N in N**, the open mapping theorem, the conjugate of an operator. (Chapter 9 of Text book)

Unit-III

Hilbert spaces: Definition and some simple properties, orthogonal complements, orthonormal sets, the conjugate space H* (Section 52-55 of Textbook)

Unit-IV

Operators in Hilbert space: Adjoint of an operator, Self-adjoint operators, Normal and Unitary operators, Projections. (Section 56-59 of Text book)

Unit-V

Finite-Dimensional Spectral Theory: Matrices, Determinants and the Spectrum of an operator, Spectral theorem, a survey of the situation. (Chapters 11 of Text book)

Text Book: G.F. Simmons – Mc Graw Hill, Introduction to Topology and Modern Analysis.

M.Sc Degree Examination

Fourth Semester

Applied Mathematics

21AM401: Functional Analysis

(Effective from the admitted batch of 2021-2022)

Time: 3 hours Maximum: 80 marks

Answer one question from each unit. All questions carry equal marks

UNIT-I

1. (a) State and prove Heine-Borel theorem.
   (b) Define topological space. Let $X = \{a, b, c\}$, construct five different topologies on X. Then find the weakest and strongest topology from them.

OR

2. (a)State and prove Tychonoff's theorem in a topological space.
   (b)Define compact space. Prove that the image of a compact space under a continuous map is compact.

UNIT-II

3. (a)State and prove Hahn-Banach theorem.
   (b) Define Banach space. Prove that $\mathbb{R}^n$ is a Banach space.

OR

4. (a) If B and B' are Banach spaces and if T is a continuous linear transformation of B onto B', then prove that T is a open map.
   (b) State and prove the uniform boundedness theorem in a Banach space.

UNIT-III

5. (a) Prove that a closed convex subset C of a Hilbert space H contains a unique vector of smallest norm.
   (b) If $\{e_i\}$ is an orthonormal set in a Hilbert space , then show that $\sum|(x, e_i)|^2 \leq \|x\|^2$ for every vector x in H.

OR

6. (a) Let H be Hilbert space, and f be an arbitrary functional in H^*. Then show that there exists a unique vector y in H such that $f(x) = < x, y >$

for every x in H.

(b) If M is a closed linear subspace of a Hilbert space H, then show that $H = M \oplus M^{\perp}$.

UNIT-IV

7. (a) If N_1 and N_2 are normal operators on H with either commutes with the adjoint of the other, then show that $N_1 + N_2$ and $N_1 N_2$ are normal.

(b) If T is an operator on H for which $(Tx, x) = 0$ for all x, then show that $T = 0$.

OR

8. (a) Show that an operator T on H is unitary $\Leftrightarrow$ it is an isometric isomorphism of H onto itself.

(b) If P is a projection on H with range M and null space N, then show that $M \perp N \Leftrightarrow$ P is self adjoint and $N = M^{\perp}$.

UNIT-V

9. (a) If T is normal, then show that the M_i's span H.

(b) Let B be a basis for H, and T an operator whose matrix relative to B is $[\alpha_{ij}]$. Then show that T is non-singular $\Leftrightarrow$ $[\alpha_{ij}]$ is non-singular and $[\alpha_{ij}]^{-1} = [T^{-1}]$.

OR

10.(a) Let T be an operator on H. If B and B' are bases for H then show that the matrices $[\alpha_{ij}]$ and $[\beta_{ij}]$ of T relative to B and B' have the same determinant .

(b) If T is normal, then show that x is an eigenvector of T with eigenvalue $\boldsymbol{\lambda} \Leftrightarrow x$ is an eigenvector of T^* with eigenvalue $\bar{\lambda}$.

M.Sc. APPLIED MATHEMATICS (FOURTH SEMESTER)

21AM 402: THEORY OF AUTOMATA AND FORMAL LANGUAGES
(With effect from 2021-2022 Admitted Batches)

Course Code:21AM402

(A total of Ten questions to be set by selecting two questions from each unit with internal choice. Each question carries 16 marks.)

Course Outcome(COs):

CO1: The various types of finite automatons like deterministic non deterministic finite automation and conversion from DFA to NDFA and Finite automations with outputs and their classification and relations , the minimization of finite automaton will taught extensively.

CO2: Classification of formal languages and relation between automaton and languages will taught.

CO3: The regular grammar and various results on these will be discussed.

CO4: The context free grammar and various results on these will be discussed.

CO5: By introducing the memory in to FA the more powerful automaton namely Turing machine and construction of Turing machines will be taught.

Course Specific Outcome(CSOs):

CSO1: To understand and utilize the abstract automatons in developing the logics for certain computations.

CSO2: To familiar with the relations between Automatons , Grammars and Languages.

CSO3: Learn to construct most powerful automaton developed by Turing machine, Universal Turing Machine and certain properties.

Learning Outcome(LOs):

LO1: The learners are familiar with finite automaton definitions, operations and various types of finite automatons like DFA , NDFA and FA's with outputs and conversions among them and constructing FA's various computations and reducing the number of states in FA.

LO2: The learners are also familiar with formal languages and classification of grammars , to find language generated by grammar and construction of grammars to generate required language.

LO3: The various concepts in Regular grammar and to prove certain sets are not regular are well versed .

LO4: Familiar with the context free grammar and its left/right derivations ,parse trees and reducing the CFG's.

LO5: Familiar with Turing machines, operations and various types of representation of TM , operations in TM and construction of TM.

Mapping of course outcomes with the program outcomes

	PO1	PO2	PO3	PO4	PO5
CO1	✓	✓	✓	✓	✓
CO2	✓	✓	✓	✓	✓
CO3	✓	✓	✓	✓	✓
CO4	✓	✓	✓	✓	

CO5	✓	✓	✓	✓	

Unit-I

The Theory of Automata: Definition of an automata, Description of a Finite Automation, Transition Systems, Properties of Transition Functions, Acceptability of a string by a finite Automation, Non Deterministic finite State Machines, The Equivalence of DFA and NDFA, Mealy and Moore models.

Unit-II

Minimization of Finite Automaton.

Formal Languages: Basic definitions and examples, Chomsky classification of Languages, Languages and their relation, Languages and Automaton.

Unit-III

Regular sets and Regular Grammars: regular expressions, Finite Automata and regular expressions, Pumping lemma for Regular sets, Application of Pumping lemma.

Unit-IV

Context-free Languages: Context-free languages and derivation trees, Ambiguity in Context-free Grammars, Simplification of Context-free Grammars, Normal forms for Context-free Grammars.

Unit- V

Turing Machines: Turing Machine model, Representation of Turing Machines, Languages Acceptability by Turing Machines, Design of Turing Machines, Universal Turing Machines and other modifications, Halting Problems of Turing Machines, unsolvable problems, the post correspondence problem.

Text book: Theory of Computer Science (Automata, Languages and Computation)

Chapters: 2,3,4,5.1 to 5.4 and 7.1 to7.5 ,7.9.3 By K.L.P. Mishra,

N. Chandrasekharan, PHI, Second edition

M.Sc Degree Examination
Fourth Semester
Applied Mathematics
21AM402:Finite Automata and Formal Languages
(Effective from the admitted batch of 2021-2022)

Time: Three hours　　　　　　　　　　　　　　　Maximum: 80 marks

Answer one question from each unit. All questions carry equal marks

Unit-I

1. (a) Construct a NDFA accepting $\{ab, ba\}$ and use it to find a DFA accepting the same set.

(b) Construct a Moore machine equivalent to Mealy machine..

Present State	Next State			
	$a = 0$		$a = 1$	
	State	output	state	output
$\rightarrow q_1$	q_3	0	q_2	0
q_2	q_1	1	q_4	0
q_3	q_2	1	q_1	1
q_4	q_4	1	q_3	0

(or)

2 (a) Find a deterministic acceptor equivalent to $M = (\{q_0,\ q_1, q_2\},\ \{a, b\}, \delta, q_{0,}\{q_2\})$ where δ is

State/Σ	a	b
$\rightarrow q_0$	q_0, q_1	q_2
q_1	q_0	q_1
q_2		q_0, q_1

(b) For every NDFA there exists a DFA which simulates the behavior of NDFA.

Unit-II

3 (a) Construct the minimum state automaton equivalent to a given automation M whose transition table is given below.

States\Inputs	a	B
$\rightarrow q_0$	q_1	q_0
q_1	q_0	q_2
q_2	q_3	q_1

q_3	q_3	q_0
q_4	q_3	q_5
q_5	q_6	q_4
q_6	q_5	q_6
q_7	q_6	q_3

(or)

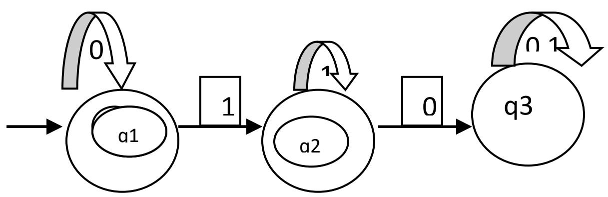
4 (a) Find the language generated by the grammar, $S \to 0S1/0A1,\ A \to 1A/1$.

(b) Find a grammar generating $L = \{a^n b^n c^i\ /n \geq 1, i \geq 0\}$.

Unit-III

5.(a) Prove that every regular expression R can be recognized by a transition system.

(b) Describe in English the set accepted by FA whose transition diagram is



(or)

6(a) Construct an FA equivalent to the regular expression, $(0+1)^*\ (00+11)\ (0+1)^*$.

(b) State and prove Pumping lemma for regular sets.

Unit-IV

7 (a) Reduce the following grammar G to CNF. Where G is $S \to aAD, A \to \frac{aB}{bAB},$

$B \to\ b, D \to d.$

(b) Consider the grammar G whose production are $S \to aS/AB, A \to \Lambda, B \to \Lambda, D \to b.$

Construct a grammar G_1 without null productions generating $L(G) - \{\Lambda\}$.

(or)

8(a) Define CNF for content free grammar and find a grammar in CNF equivalent to

$$S \to aAbB,\ A \to aA/a,\ B \to bB/\ b.$$

(b) Let G be $S \to AB,\ A \to a, B \to C/b\ , C \to D, D \to F, E \to a$. Eliminate unit products and get an equivalent grammar.

Unit-V

9. (a) What are the methods to represent Turing machines?

(b) Design a Turing machine to accept palindromes over $\{a, b\}$.

(or)

10(a) Design a Turing machine that accepts $\{0^n 1^n\ /n \geq 1\}$.

(b) Explain about Halting problems in Turing machine.

M.SC. APPLIED MATHEMATICS (FOURTH SEMESTER)
21AM 404(A): BOUNDARY VALUE PROBLEMS-II
(With effect from 2021-2022 Admitted Batch)
Course Code:21AM404(A)

(A total of Ten questions to be set by selecting two questions from each unit with internal choice. Each question carries 16 marks.

Course Outcome(COs):

CO1: The qualitative properties of differential equations like local stability, various types of stability of the systems and necessary and sufficient conditions linear, weakly non linear systems and two dimensional systems are taught.

CO2: The global stability of the systems by various methods with certain applications are taught.

CO3: Certain mathematical models for one species and two species models and their local and global stability of the positive equilibrium points are taught.

CO4: Analysis and methods of non linear systems , existence of solutions , certain differential inequalities and non linear variation of parameters formula are taught.

CO5: The qualitative property oscillatory and non oscillatory properties of second order equations are taught.

Course Specific Outcome:

CSO1: Comparing the quantitative and qualitative properties of linear and weakly non-linear systems.

CSO2: Formulation of Mathematical models for population dynamics and discussing the qualitative properties of positive equilibrium points of the systems.

CSO3: Understanding the methods and analysis of non-linear systems and certain results on Oscillatory theory.

Learning Outcome:

LO1: The learner must familiar with stability, asymptotically stability and unstable concepts. The learner also familiar with various stability properties and necessary and sufficient conditions for local stability analysis.

LO2: The concepts on stability based on Liapunov second method should applied to various problems.

LO3: The mathematical modeling on single and multi species are formulated and learner are familiar with to test their local and global stability of the positive equilibrium point of the system.

LO4: With out assuming the Lipschitz condition the learner establish the existence of solutions of system of initial value problems and certain properties of the system are well versed.

LO5: Results on Oscillatory and non oscillatory properties of the differential equations are to be familiar.

Mapping of course outcomes with the program outcomes

	PO1	PO2	PO3	PO4	PO5
CO1	✓	✓	✓	✓	
CO2	✓	✓	✓	✓	
CO3	✓	✓	✓	✓	
CO4	✓	✓	✓	✓	
CO5	✓	✓	✓	✓	

Unit-I

Stability of linear and weakly non-linear systems, continuous dependence and stability properties of linear, non-linear and weakly non-linear systems. Two dimensional systems. (chapter III of text book-1)

Unit-II

Stability by Liapunov second method, Autonomous systems, quadratic forms, Krasovski's Method. Construction of Liapunov functions for linear systems with constant coefficients. Selection of total energy function as a Liapunov Function, Stability based on first approximation (Chapter V of text book-1)

Unit-III

Mathematical Models in Population Dynamics: Introduction, single species Models, Two species Lotka volterra Models, Multi species Models. (chapter VI of text book-1)

Unit-IV

Analysis and Methods of non-linear differential equations, Existence theorem, extremal solutions, upper and lower solutions. Existence via upper and lower solutions.(Sec. 6.1-6.4 in Chapter VI of text book-2)

Bihari's inequality, Application of Bihari's integral inequality. Non-linear variation of parameters formula Alekseev's formula. (Sec. 6.1 to 6.4,6.6-6.7 in Chapter VI of text book-2)

Unit-V

Oscillations of second order equation, sturms comparison theorems, Elementary linear Oscillations, comparison theorem of Hille-Winter. (Chapter VIII of text book-2)

Text Books:

1. M.Rama Mohan Rao, Ordinary Differential equations, Theory methods and applications, Affiliated East-West Press Pvt.Ltd., New Delhi. (1980).
2. V.Lakshmikantam, S.G.Deo and V.Raghavendra, Text book of ordinary differential equations (second edition) Tata Mc Graw Hill, New Delhi. (1997).

M.Sc Degree Examination
Fourth Semester
Applied Mathematics
21AM404(A):Boundary Value problems-II
(Effective from the admitted batch of 2021-2022)

Time: Three hours Maximum: 80 marks

Answer one question from each unit. All questions carry equal marks.

Unit-I

1. (a) Prove that the system $x' = A(t)x$ is stable if and only if its fundamental matrix is bounded.

(b) Show that the corresponding system for $u'' + \frac{2u'}{t+1} = 0$ is stable but not uniformly stable.

(or)

2. (a) Prove that the solution $x' = A(t)x$ is strongly stable if and only if there exists a positive constant M such that $\|\Phi(t)\| \le M$, $\|\Phi^{-1}(t)\| \le M$, for , $t \ge t_0$.

(b) Discuss the solutions of the $u'' + 2ku' + q^2u = 0$, where k and q are positive constants, with reference to nature of the critical point (0, 0) of its two dimensional system.

Unit-II

3. (a) Construct a Liapunov function for the system $x' = Ax$ where

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}.$$

(b) Determine the type of stability of the critical point (0, 0) of the following linear system and sketch the phase portraits $x' = Ax$, $A = \begin{pmatrix} -3 & 1 \\ 4 & -2 \end{pmatrix}$.

(or)

4.(a) Explain Krasovskii's method and determined the stability of the zero solution of

$$x_1' = -x_1 - x_2 - x_1^3,\ x_2' = x_1 - x_2 - x_2^3.$$

(b) Select a suitable Liapunov function and show that the critical point (0, 0) of

$$x_1' = -2x_1 + 5x_2 + x_2^2, \qquad x_2' = -4x_1 - 2x_2 + x_1^2$$

Unit-III

5.(a) If $\frac{a}{d} >$ $x_1' = x_1(a - bx_1 - cx_2)$, $x_2' = x_2(-d + ex_1 - fx_2)\frac{b}{e}$ holds, then the unique positive equilibrium $E_4(x_1^*, x_2^*)$ of

$$x_1' = x_1(a - bx_1 - cx_2), \qquad x_2' = x_2(-d + ex_1 - fx_2),$$

where a, b, c, d, e and f are positive constants, is globally asymptotically stable.

(b) Show that the positive equilibrium of the prey-predator system

$$x_1' = x_1(4 - 2x_1 - 3x_2),\ x_2' = x_2(-6 + 4x_1 - 10x_2)$$

is globally asymptotically stable.

(or)

6. (a) If there exists $\eta_i, i = 1,2$ such that $b_{ii} > \frac{1}{2}\sum_{j=1, j\ne i}^{2} (b_{ij} + b_{ji}) + \eta_i, i = 1,2,$

$$\frac{b_{12}}{b_{22}} < \frac{a_1}{a_2} \text{ and } \frac{b_{11}}{b_{21}} < \frac{a_1}{a_2}$$

hold. Then, the nonnegative equilibrium $E_2(a_1/b_{11}, 0)$ of

$$x_1' = x_1(a_1 - b_{11}x_1 - b_{12}x_2),\ x_2' = x_2(a_2 - b_{21}x_1 - b_{22}x_2),$$

where a_i, b_{ij} $(i, j = 1,2)$ are positive constants, is asymptotically stable.

(b) Find the solution of logistic equation $p' = p(a - bp)$, $p(0) = p_0$.

Unit-IV

7. (a) State and prove existence theorem for the solution for the initial value problem.

(b) State and prove Bihari's inequality .

(or)

8. (a) Define upper and lower solutions and find upper and lower solutions of $x' = x^2$, $x(0) = -1$.

(b) Show that the solution of IVP , $x' = 2tx^2, x(t_0) = x_{0,}\ t \geq t_0 \geq 0$ is $x(t, t_0, x_0) = [(t_0{}^2 + \frac{1}{x_0}) - t^2]^{-1}$.

Unit-V

9.(a) Check the equation $u'' + u = 0$ is oscillatory or not.

(b) Establish the normal form of the Bessel's equation $t^2x'' + tx' + (t^2 - p^2)x = 0$.
Further,

(i) Show that the solutions $J_p(t)$ of Bessel's equation and $Y_p(t)$ of normal form equation have common zeros for *t >0.*

(ii) If $0 \leq p < 1/2$, show that every interval of length π contains at least one zero of $J_p(t)$.

(or)

10 (a) State and prove Sturm's comparison theorem.

(b) Let $r(t)$ be a continuous function (for $t \geq 0$) such that $r(t) > m^2 > 0$, where m is an integer.

Consider the equations $x'' + m^2x = 0$, $y'' + r(t)y = 0$, $t > 0$. If $y(t)$ is any solution of the second equation prove that $y(t)$ must vanish in any interval of length $\frac{\pi}{m}$.

M.Sc. APPLIED MATHEMATICS(FOURTH SEMESTER)

21AM 405(B): OPTIMIZATION TECHNIQUES-II

(With effect from 2021-2022 Admitted Batch)

Course Code:21AM405(B)

(A total of ten questions are to be set taking two questions from each unit with internal choice in each unit. Each question carries 16 marks.)

Course Outcome(COs):

CO1: This course introduces some key concepts of optimization techniques.

CO2: Provides an in-depth knowledge of problem solving through various optimization techniques.

CO3: Tests and develops the students' knowledge of basic understanding of the problems through practical illustrations and examples.

CO4: Improves the logical thinking ability of the students and helps gain access to various employment opportunities.

CO5: Infuses practical knowledge that helps in pursuing higher studies as well as getting employment.

Course Specific Outcome(CSOs):

CSO1: Study on some optimization techniques like Game theory, job sequencing techniques, exposure to inventory management and Replacement problems, their solution techniques in real world problems.

CSO2: Introducing the concepts of Queueing models and their solutions.

CSO3: Gain knowledge of finding critical paths in network analysis. These concepts of optimization have been further expanded to study Network analysis and scheduling techniques.

Learning Outcome(LOs):

LO1: After studying the students are expected to learn some techniques of optimization.

LO2: Gain ability to problem formulations, analysis and solution techniques.

LO3: Improves the logical thinking ability.

LO4: Provides an in-depth knowledge of problem solving through various optimization techniques.

LO5: Gain knowledge in computational skills and numerical problem solving.

Mapping of course outcomes with the program outcomes

	PO1	PO2	PO3	PO4	PO5
CO1	✓	✓		✓	
CO2			✓		
CO3		✓	✓		
CO4	✓			✓	✓
CO5		✓		✓	✓

Unit-I

Game Theory, Solution of Games with and without saddle points, minimax / maximini principle, principle of Dominance, matrix method for (m X n) Games without saddle point, algebraic method.**(Chapter 1 of Unit 4(except 1.22)**
Jog Sequencing: Processing of n-jobs through 2/3/m$^{machines(}$**Chapter 6 of unit 4)**

Unit-II

Inventory, classification inventory models, EOQ models with and without shortages, multi item deterministic models, dynamic demand Models. **(Chapter 2 of unit 4 (2.1 to 2.17))**

Unit-III

Replacement Models: Replacement of items that deteriates with time, individual replacement. Group replacement policies, recruitment and production problem.Equipment and renewal problem systems reliability.**(Chapter 4 of unit 4)**

Unit-IV

Queuing theory: distribution in queuing systems, Poisson process. Classification and solutions of Queuing model, models 1-4 **(Chapter 5 of unit 4) (5.1 to 5.15)**

Unit-V

Network analysis, PERT/ CPM Techniques network diagram representation time estimates and critical path in net work analysis, uses of PERT / CPM Techniques **(Chapter7 of unit 4)**

Text book: Operations Research by S.D.Sarma (12 th Edition), KedarnathRamnath and company.

M.Sc Degree Examination
Fourth Semester
Applied Mathematics
21AM405(B): Optimization Techniques-II
(Effective from the admitted batch of 2021-2022)

Time: 3 hours Maximum: 80 marks

Answer one question from each unit. All questions carry equal marks

UNIT-I

1) a) Explain the principle and rules of dominance to reduce the size of the payoff matrix.

b) Solve the following game whose payoff matrix is given below:

	B_1	B_2	B_3	B_4
A_1	5	-10	9	0
A_2	6	7	8	1
A_3	8	7	15	1
A_4	3	4	-1	4

OR

2) a) Find the sequence that minimizes the total time required in performing the following jobs on three machines A, B, and C in the order ABC. Processing times (in hours) are given below:

Job:	1	2	3	4	5
Machine A:	8	10	6	7	11
Machine B:	5	6	2	3	4
Machine C:	4	9	8	6	5

b) Explain algebraic method for solving a game problem.

UNIT-II

3) a) Derive the optimal lot size formula for the inventory problem with constant demand, no shortages and finite rate of replenishment.

b) An item is produced at the rate of 128 units per day. The annual demand is 64 units. The set-up cost for each production run is Rs. 24 and inventory carrying cost is Rs. 3 per unit per year. There are 250 working days for production each year. Develop an inventory policy for this item.

OR

4) a) Obtain an expression for the economic order quantity for an inventory control model with gradual supply and shortages allowed.

b) Find the optimum order quantity of a product for which the price breaks are as follows:

Range of quantity	Purchase price per unit(in Rs.)
$0 < q_1 < 800$	1.00
$800 \leq q_2$	0.98

The yearly demand for the product is 1600 units, the cost of storage is 10% per year, and the cost of ordering is Rs.6.

UNIT-III

5) a) Discuss the policy of replacement of items whose maintenance cost increases with time and the value of money remains constant during the period.

b) A firm is considering replacement of a machine which is priced at Rs. 60,000 and running costs are estimated at Rs. 6,000 for each of the first four years, increasing by Rs. 2,000 per year in the fifth and subsequent years. If money is worth 10% per year, determine when should the machine be replaced?

OR

6) a) Explain clearly with suitable examples the different costs that are involved in the inventory problem. What constitute the ordering and carrying costs?

b) A computer contains 10000 resistors. When any register fails, it is replaced. The cost of replacing a register individually is rupees 1 only. If all the registers are replaced at the same time, the cost per register would be reduced to 35 paise. The percentage of surviving registers say, at the end of month and the probability of failure during the month are given below. What is the optimal replacement plan?

t	0	1	2	3	4	5	6
$S(t)$	100	97	90	70	30	15	0
$P(t)$	--	0.03	0.07	0.20	0.40	0.15	0.15

UNIT-IV

7) a) For the queueing model $(M/M/1):(N/FCFS)$, obtain the steady-state probability expression for p_n. Find the probability that the system is empty, and the expected queue length.

b) Consider a self-service store with one cashier. Assume Poisson arrivals and exponential service times. Suppose that 9 customers arrive on the average every 5 minutes and the cashier can serve 10 in 5 minutes. Find

(i) the average number of customers queuing for service.

(ii) the probability of having more than 10 customers in the system.

OR

8) a) Explain$\mathrm{M/M/C}$queue model and obtain the steady-state probability expression for $\mathrm{p_n}$.

b) A Supermarket has two girls serving at the counters. The customers arrive in a Poisson fashion at the rate of 12 per hour. The service time for each customer is exponential with mean 6 minutes. Find

I. The probability that an arriving customer has to wait for service

II. Average number of customers in the system, and

III. The average time spent by a customer in the supermarket.

UNIT-V

9) The following table lists the jobs of a network along with their time estimates

Job	Duration in days		
	Optimistic	Most likely	Pessimistic

1-2	3	6	15
1-6	2	5	14
2-3	6	12	30
2-4	2	5	8
3-5	5	11	17
4-5	3	6	15
6-7	3	9	27
5-8	1	4	7
7-8	4	19	28

(i) Draw the project network.

(ii) Calculate the length and variance of the critical path; and

(iii) What is the approximate probability that the jobs on the critical path will be completed in forty one days?

OR

10) Draw a network diagram for a project consisting of 7 tasks $(A, B, \ldots, G)$ in which the following precedence relationship must hold ($X < Y$ means X must be completed before Y can start):
$A < C; A < B; B < D; B < G; C < D; C < G; D < E; E < F$. Given the following task times for the above project, locate the critical path and find the free float and total floats for the non-critical activities.

Task:	A	B	C	D	E	F	G
Time:	30	7	10	14	10	7	21

M.Sc. APPLIED MATHEMATICS (FOURTH SEMESTER)

21AM 406(C):RELATIVITY AND COSMOLOGY-II

(With effect from 2021-2022 Admitted Batch)

Course Code:21AM406(C)

(A total of Ten questions to be set by selecting two questions from each unit with internal choice. Each question carries 16 marks.)

Course Outcome(COs): At the end of the course, the students will be able to

CO1: Describe the basic concepts of the theory of relativity.

CO2: Explain the concept of invariance.

CO3: Explain the concept of space-time.

CO4: Discuss the equivalence principle and explain the twin paradox.

CO5: Describe gravity as space-time curvature. Also, describe general theory of relativity as mathematical basis of physical cosmology.

Course Specific Outcomes(CSOs):

CSO1: The students know how Einstein used the Equivalence Principle to extract some general relativity results (gravitational redshift, time dilation, and light deflection) and in turn how such physics led him to the idea that gravitational field is simply a curved spacetime.

CSO2: The students gain knowledge of the different cosmological parameters for understanding the observed universe.

CSO3: The students learn many features of the universe which are not understood by Special Theory of Relativity and Classical Mechanics.

Learning Outcomes(LOs): After completing the course, you:

LO1: master the equivalence principle and have a good knowledge of how this leads to a geometric description of gravity, in the form of the general theory of gravity.

LO2: have detailed knowledge about how space and time are curved for spherically symmetric mass distributions**.**

LO3: have acquired basic knowledge about the cosmological concordance model, and how it is based on Einstein's theory of gravity.

LO4: are able to present complex topics in general relativity in a clear and pedagogic way, and communicate this to fellow students.

LO5: can communicate the basic principles behind the theory, as well as tests of the theory, to people outside the community.

Mapping of course outcomes with the program outcomes					
	PO1	**PO2**	**PO3**	**PO4**	**PO5**
CO1	√	-	√	√	-
CO2	√	-	√	√	-
CO3	√	-	√	√	-
CO4	√	-	√	√	-
CO5	√	-	√	√	-

Unit-I

The Fundamental Principles of General Relativity, Principle of Covariance, Principle of equivalence, Principle of Mach, Gravitational field in empty space, Gravitational field in the presence of matter and energy. Simple consequences of principle of equivalence, Newton's theory as a first approximation. The Schwarzschild line element, the three crucial tests of Relativity(Chapter VII-Articles 94-99 of Text book).

Unit-II

Line elements for systems with spherical symmetry, static line element with spherical symmetry, Schwarzschild exterior and interior solutions, Non-static line elements with spherical symmetry-Birkhoff's theorem. The generalized Lorentz Electron theory the field equations. The gravitational field of a charged particle (Chapter VII-Articles 102 &107 of Text book).

Unit-III

The Energy-Momentum Tensor for Disordered Radiation. The Gravitational Mass of Disordered Radiation. The Energy-Momentum Tensor Corresponding to a Directed Flow of Radiation. The Gravitational Field Corresponding to a Directed Flow of Radiation. The Gravitational Action of a Pencil of Light. The Gravitational Action of a Pulse of Light. Discussion of the Gravitational Interaction of Light Rays and Particles. The Generalized Doppler Effect(Chapter VIII-Articles 120 &127 of Text book).

Unit-IV

Application of general relativity to cosmology, The three possibilities for a homogeneous static universe, The Einstein line element, the de-sitter line element, Special relativity line element, The geometry of the Einstein universe, Density and pressure of material in Einstein universe. Behavior of test particles and light rays in the Einstein universe (Chapter X-Articles 133-139 of Text book).

Unit-V

Comparison of Einstein model with actual universe, Geometry of the de-sitter universe, Absence of matter and radiation from de-sitter universe, Behavior of test particles and light rays in the de-sitter universe(Chapter X-Articles 140-144 of Text book).

Text Book:

Relativity, Thermodynamics and Cosmology, R.C. Tolman, Clarendon Press, Oxford.

Books for reference:

1. Theory of Relativity by S.R. Roy and Raj Bali, Jaipur Publishing House, Jaipur.

M.Sc Degree Examination

Fourth Semester

Applied Mathematics

21AM406(C): Relativity & Cosmology

(Effective from the admitted batch of 2021-2022)

Time: 3 hours Maximum: 80 marks

Answer one question from each unit. All questions carry equal marks

Unit-I

1). (a) Explain the Mach's principle.

(b) Discuss the following concepts:

(i) The advance of perihilion of mercury,

(ii) Gravitational deflection of light rays.

(OR)

2). (a) Discuss about the principle of Equivalence.

(b) Show that the relativistic orbit of a planet round the Sun is given by $\frac{d^2u}{d\emptyset^2} + u = \frac{m}{h^2} + 3mu^2$,

where $r^2 \frac{d\emptyset}{ds} = h$.

Unit-II

3). (a) Obtain Schwarzchild's interior solution.

(b) State and prove Birkhoff's theorem.

(OR)

4). (a) Obtain Schwarzchild's exterior solution of an isolated gravitating body.

(b) Discuss Reissner-Nordstrom solution at a charged particle.

Unit-III

5). (a) Obtain energy momentum tensor for disordered radiation.

(b) Discuss energy-momentum tensor corresponding to a directed flow of radiation.

(OR)

6). Explain the following concepts:

(i) The gravitational action of a pencil of light.

(ii) The gravitational action of a pulse of light.

Unit-IV

7) In the static cosmological model, derive Einstein's line-element and discuss the geometrical properties in

Einstein's Universe.

(OR)

8) Discuss the behaviour of motion of test particles and Doppler effect in Einstein Universe.

Unit-V

9) In the static cosmological model derive De-Sitter line-element and discuss the geometrical properties.

(OR)

10) Discuss the behaviour of the particles and Doppler effect in De-Sitter Universe.

M.Sc. APPLIED MATHEMATICS (FOURTH SEMESTER)

21AM 407(D): Numerical solution of Partial Differential Equations-II

(With effect from 2021-2022 Admitted Batch)

Course Code:21AM407(D)

(A total of Ten questions to be set by selecting two questions from each unit with internal choice. Each question carries 16 marks.)

Course Outcomes(COs):

On completion of this course, the students will have the ability to

CO1 :Understand basic theory, importance, efficiency of finite element methods.

C02: solve elliptic partial differential equations using weak formulations and finite element Methods.

C03 :construct various finite elements to approximate the solution of PDEs.

C04 :analyses the convergence analysis to identify the robustness and possible improvement of the finite element methods.

C05: implement finite element formulation of various partial differential equations.

Course Specific outcomes (CSOs):

CSO1: Learn about solution of ODEs numerically stable schemes.

CSO2: Learn about Parabolic PDEs and finding its solution numerically.

CSO3: Learn about Elliptic PDEs and finding its solution numerically

Learning outcome(LOs):

LO1: At the end of the course, a student will be able to explain the concepts and principles used in the formulation and application of the finite element method.

LO2: demonstrate an ability to formulate, implement, and document solutions to solve simple engineering problems using the finite element method.

LO3: Learn about the function spaces.

LO4: Interpolation and polynomial approximation in finite element spaces.

LO5: They will be able to construct stable finite element methods for solving problems in science, engineering, option pricing.

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Mapping of course outcomes with the program outcomes:

	PO1	PO2	PO3	PO4	PO5
CO1	✓	✓	✓	✓	✓
CO2	✓	✓	✓	✓	✓
CO3	✓	✓	✓	✓	✓
CO4	✓	✓	✓	✓	✓
CO5	✓	✓	✓	✓	✓

Unit-I

Test functions and Distributions, operations with distributions, support of distributions, convolution of distributions, definition and basic properties. (Section 1.1, 1.2, 1.3, 1.5, 1.6 of Text book 1).

Unit-II

Variational problem, variational formulation of one-dimensional model problem, FEM for the model problem with piecewise linear functions, Ritz-Galerkin method, Error estimate for the model problem, FEM for the Poisson equation, Geometrical interpretation of FEM, Hilbert spaces, A Neumann problem: Natural and essential boundary conditions. (Section 1.1 to 1.7 of Textbook 2)

Unit-III

Abstract formulation of FEM for Elliptic problems: The continuous problem, Lax-Milgram theorem, stability estimate, discretization, error estimates, energy norm. (Chapter 2, of Textbook 2)

Unit-IV

Finite element spaces: Regularity requirements, Examples of finite elements. Approximation theory for FEM. Error estimates for elliptic problems: introduction, interpolation with piecewise linear functions in two dimensions, error estimates for FEM for elliptic problems, Regularity of exact solution, Error estimate in L2 norm, (chapter 3, 4 of Textbook 2)

Unit-V

FEM for Parabolic Problems, One Dimensional model problem, semi discrete and fully discrete Scheme, Error estimates. (Chapter 8 of Textbook 2)

Text Books:

1. S Kesavan, Topics in functional analysis and applications, new age international limited, 2003.
2. Claes Johnson, Numerical solutions of partial differential equations by the finite element method, Cambridge university press, Cambridge, 1987.

Reference Books:

1. S.C Brenner, R. Scott, The mathematical theory of finite element methods, Springer, New York, 2008.
2. J.N. Reddy, An Introduction to the Finite Element Method, Tata McGraw-Hill Edition.